Geomathematical Modeling of Magma Crystallization: Linking Mineral Formation to Geochemical Processes in Volcanic Environments

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Abstract

Mathematically, rocks can be modeled using the parametric materials (Minerals) that constitute the rocks. These materials (Minerals) depend on the variability of temperature (T), under isobaric condition and the bulk composition(X), of the parent (Basalt) magma and then analysed it using certain mathematical parametres such as Matrix, set Notation and Geometry. Ten samples were collected at random at different points in Kassa Volcanic Field (KVF). These samples were examined under thin section in university of Jos geology laboratory, using Petrographic Microscope both plane and cross polarized light and the detailed minerals observed were, Olivine, Pyroxene, Plagioclase and magnetite. Two kilograms(2kg) of each of the samples were pulverized and subjected to chemical analysis using Inductively couple Plasma Mass spectrometer (ICPMS) in Canadian Geochemical Laboratory and the chemical compositions analysed were SiO_2, Na_2O, K_2O, Fe_2O_3, FeO, MgO, CaO, TiO_2, etc. For trace elements, it includes Ba, Rb, Sr, Cs, Ni, Cr, V, U, Th, Nb, Li, Ga, Zr, Mn, etc. In modeling of basalt and its equivalent, granite, mathematical computation, examination of minerals under thin section, chemical analysis of the basalt were the major objectives of this research After all these analyses using mathematical modeling approach, three novel empirical parametres: [Matrix, Set notation and Geometry] were developed and employed to study the essential rock forming minerals, the distribution of trace elements in rock as well as establishing a relationship between continuous and discontinuous reaction series.

Keywords

Application, Mathematical, Modeling, Matrix, Set Notation, Coordinate Geometry

1. Introduction

Mathematical models involve the use of equations consisting of variables and constants according to Hubbert [1]. Geomathematics, in its broadest sense includes all applications of mathematics to studies of the earth's crust.

In practice, the resulting mathematical model may then consist of fitting to the data a few linear terms out of a Taylor expansion [2] for the mainly unknown geological process. Quantitative geologic considerations are indispensible, first for selection of the variables to be measured, for designing the mathematical model and later for interpretation and evaluation of the results.

Mathematical equations were applied to rock forming minerals during crystallization of magma from the beginning of crystallization to the end of crystallization to produce the "rock model".

Both olivine and pyroxene are silicates that contain a variable amount of magnesium (Mg^{2+}) and iron (Fe^{2+}). Pure magnesium olivine that does not contain iron is forsterite. The other end member of the olivine series with iron instead of magnesium is called fayalite. Chemical expressions for olivine are $(Mg, Fe)_2\ SiO_4$ or $(Mg_{x_0}\ Fe_{1-x_0})_2\ SiO_4$ where x_0 denotes the fraction of Mg atoms in olivine. The variable x_0 then is equivalent to our variable y_1 which is measured in percent. When $x_0=1 (or\ y_1=100)$, the formula becomes Mg_2SiO_4 for forsterite. Orthopyroxene has chemical composition $\left(Mg_{x_p}Fe_{1-x_p}\right)$ SiO_3. When $x_p=1(y_2=100)$, we have the end member enstatite; when $x_p=0$, the pyroxene is called ferrosilite. In a system where olivine and pyroxene coexist, a chemical process may take place in that some Fe^{2+} ions go from olivine to pyroxene or vice versa. Simultaneously, the same quantity of Mg^{2+} ions goes in the opposite direction. This reaction can be written as:

$$\left(Mg'_{x_0}\ Fe'_{1-x_0}\right)_2 SiO_4 + 2\left(Mg'_{x_p}\ Fe'_{1-x_p}\right) SiO_3 \rightleftharpoons$$

Olivine' orthopyroxene′

$$\left(Mg''_{x_0}\ Fe''_{1-x_0}\right)_2 SiO_4 + 2\left(Mg''_{x_p}\ Fe''_{1-x_p}\right) SiO_3$$

olivine″ orthopyroxene″

The minerals at the left hand side of this reaction are indicated as olivine′ and orthopyroxene′. This implies that in a given situation (constant temperature, pressure and other variables), $x_0=x_0^{'}$ and $x_p=x_p^{'}$. If the conditions are changed we obtain olivine″ with $x_0^{'}=x_0^{''}$ and pyroxene″ with $x_p=x_p^{''}$. If the system is in equilibrium, the reaction is reversible. The above equation may be simplified to $Mg(SiO_4)\frac{1}{2}+FeSiO_3=Fe(SiO_4)\frac{1}{2}+MgSiO_3$

The functional relationship between x_0 and x_p was studied by several authors including Ramberg and DeVore (1951). It depends on several factors, the most important one being temperature. When other factors such as pressure are kept constant, the equilibrium constant K satisfies:

$$K=e^{-\Delta G/RT}$$

Where $\Delta G$the change in Gibbs free energy for the exchange reaction is in simplified form R is the gas constant and T absolute temperature

If both minerals behave as ideal solutions:

$$K=\frac{x_p}{1-x_p}\ \cdot\ \frac{1-x_0}{x_0}$$

Bowen’s uses temperatures with respect to time to establish the relationship between the essential rock forming minerals, but the major problem is that, there was no mathematical documentation that complements the explanation of his reaction model, that is there is no coordinate geometric relationship between his two reactions series that comprise

The continuous (solid solution) and Diccontinuous reaction series (fractional crystallization), as far as i have read.

The main aim of this research is the application of mathematical model to resolve the various problems related to the distribution of chemical elements and the elemental substitutions that could take place in the igneous rocks and their mineralogical components, originating from the crystallization of magma as well as establishing a geometric relationship between continuous and discontinuous reaction series that could not be completely explained by Bowen’s Model and the objective is to derive mathematical equations to solve these problems.
Application of Mathematical model is an empirical model developed with mathematical equations in this research to rock forming minerals to understand the sequence of crystallization of magma. It is a method developed as an alternative model to Bowen’s reaction model to help students of mineralogy, petrology and other courses related to crystal geology to understand the formation of rock forming during crystallization of magma. It involves mathematical equation, to know how to calculate the empirical formula of each mineral that crystallizes from the melt. It also helps students to classify, name and understand genetic origin of the minerals.

This research when completed would be an alternative model to Bowen’s reaction model and would be one of the simplified models to study mineralogy and petrology as well as its application to mineral exploration. Bowen’s uses Y-shape to explain the progressive crystallization of minerals from the melt as temperature drops, with the left side of the shape representing discontinuous reaction series and is classified as ferromagnesian, the left side of the shape representing continuous reaction series and is classified as plagioclase series and finally the tail end pointing downward representing felsic minerals. While this research uses right-angled triangle to explain the progressive crystallization of magma as temperature drops, such that the second two minerals to form representing the continuous and discontinuous reactions are perpendicular to the first two formed minerals, with the bigger triangle representing the mafic minerals and the smaller triangle representing the felsic minerals, therefore the horizontal axis representing the discontinuous reaction series and classifies it as Basic series (Magnesian) and the vertical axis representing the continuous reaction series and classifies it as ferromagnesian and plagioclase.

As crystallization of magma progresses sequentially, the compositions of the minerals under their own influence, which make up the rocks, must be unitary (the same uniform state), so that the resultant rocks must be minimum Gibbs free energy at equilibrium [3]

This research model is a spectrum that covers the range of the major minerals such as olivine, pyroxene, amphibole, mica and feldspar, and its associated rocks such as granite, basalt, andesite, trachyte etc.

All these minerals are formed during crystallization of magma as temperature drops. In order to model these rocks mathematically, mathematical equations are used and temperatures at which each mineral forms must be considered and can be measured using Geological Thermometer

2. Literature Review

Bowen’s reaction principle, first propounded in 1928 by Norman Bowen, which explains how mineral can respond to changing equilibrium conditions when a magma is cooled, by either a continuous diffusing-controlled exchange of elements with the magma or discontinuous melting of the material [4].

The periodic law was developed independently by Dmitri Mendeleev and Lothar Meyer in 1869. Mendeleev created the first periodic table and was shortly followed by Meyer [5]. They both arranged the elements by their mass and proposed that certain properties periodically reoccur. Meyer formed his periodic law based on the atomic volume or molar volume, which is the atomic mass divided by the density in solid form. Mendeleev's table is noteworthy because it exhibits mostly accurate values for atomic mass and it also contains blank spaces for unknown elements.

Goldschmidt proposed his Classical general rules to explain the distribution of the elements, in which ions of similar size and charge substitute themselves [6].

Ringwood proposed the complementary use of the concept of electronegativity in order to understand the distributions of the chemical elements that could not be explained completely with the Goldschmidtian rules, especially when the minerals being investigated had high percentages of covalent bonding [7].

Fournier and Rowe, state that silica Geothermomenter works because that solubility of the various silica minerals (Quartz, and chalcedony, S_iO_2) increase [8].

3. Material and Methods

The procedures involve thin sectioning of the rocks, using to study rocks petrologic microscope under plane and cross-polarised light, chemical analysis using ICPMS**,** and mathematical computation.

The procedures used for data collection are basically from primary source such as rock samples collected along Kassa volcanoes and secondary source, such as journals, internet, textbooks, etc. The following rock samples of gabbroic composition were collected and their compositions were used to classify them using mathematical computation.

- Basalt
- Volcanic agglomerate
- Volcanic ash

3.1 Technical Development

Application of Mathematical model is an empirical model developed in this research to rock forming minerals to understand the sequence of crystallization of magma. It is a method developed as an alternative model to Bowen’s reaction model to help students of mineralogy, petrology and other courses related to crystal geology to understand the formation of rock forming minerals during crystallization of magma. It involves mathematical equations, to know how to calculate the empirical formula of each mineral that crystallizes from the melt. It also helps students to classify, name and understand genetic origin of the minerals and rocks.

This research surgests an alternative model to Bowen’s reaction model and would be one of the simplified models to study mineralogy and petrology as well as its application to mineral exploration. Bowen’s uses Y-shape to explain the progressive crystallization of minerals from the melt as temperature drops, with the left side of the shape representing discontinuous reaction series and is classified as ferromagnesian, the right side of the shape representing continuous reaction series and is classified as plagioclase series and finally the tail end pointing downward representing felsic minerals. While this research uses right-angled triangle to explain the progressive crystallization of magma as temperature drops, such that the second two minerals to form representing the continuous and discontinuous reactions are perpendicular to the first formed mineral, with the bigger triangle representing the mafic minerals and the smaller triangle representing the felsic minerals, therefore the horizontal axis representing the discontinuous reaction series and classifies it as Basic series (Magnesian) and the vertical axis representing the continuous reaction series and classifies it as ferromagnesian and plagioclase as shown in Fig 2.

The chemical and mineralogical compositions of rocks, as well as its temperatures of crystallization under isobaric condition, can be measured, on the assumption that, rocks can be examined under thin section and geochemically analyzed, or alternatively existing rocks can be melted, and then frozen to measure its temperature or can be synthesized from chemical reagents using modern chemical knowledge and equipment. For crystal to form, the following assumptions are made during cooling of magma.

- With respect to homogenous nucleation, the growth of a mineral grain requires that the appropriate atoms and ions must find each other and then chemically bond to form what will become the nucleus of a crystal.
- The algebraic sum of anionic radical equals the charges it carries, during crystallization.
- Because the anionic radical has an electron greater than the proton, cation is needed to balance the radical so that the algebraic sum of both, the radical and the cation will be equal to zero, as the temperature drops.
- As crystallization of magma progresses sequentially, the compositions of the minerals under their own influence, which make up the rocks, must be unitary (the same uniform state), so that the resultant rocks must be minimum Gibbs free energy at equilibrium.

3.2 Conceptual Design Model

The concept of this model takes the shape of geometric right-angled triangle which indicates two series of reactions as attributed by Bowen’s: fractional crystallization and solid solution (Fig 1). To achieve this concept, geological thermometer is used to estimate the temperature at which it equilibrated in the subsurface. The following geological thermometers are used to estimate the temperatures at which rocks were formed.

• Silical (SiO_2) geothermometer: this is a battery of the rock forming mineral.
• Sodic-potassic (Na-K) geothermometer: this is an engine room of rock forming mineral.
• Sodic-potassic-calcic (Na-K-Ca) geothermometer: this is geological indicator

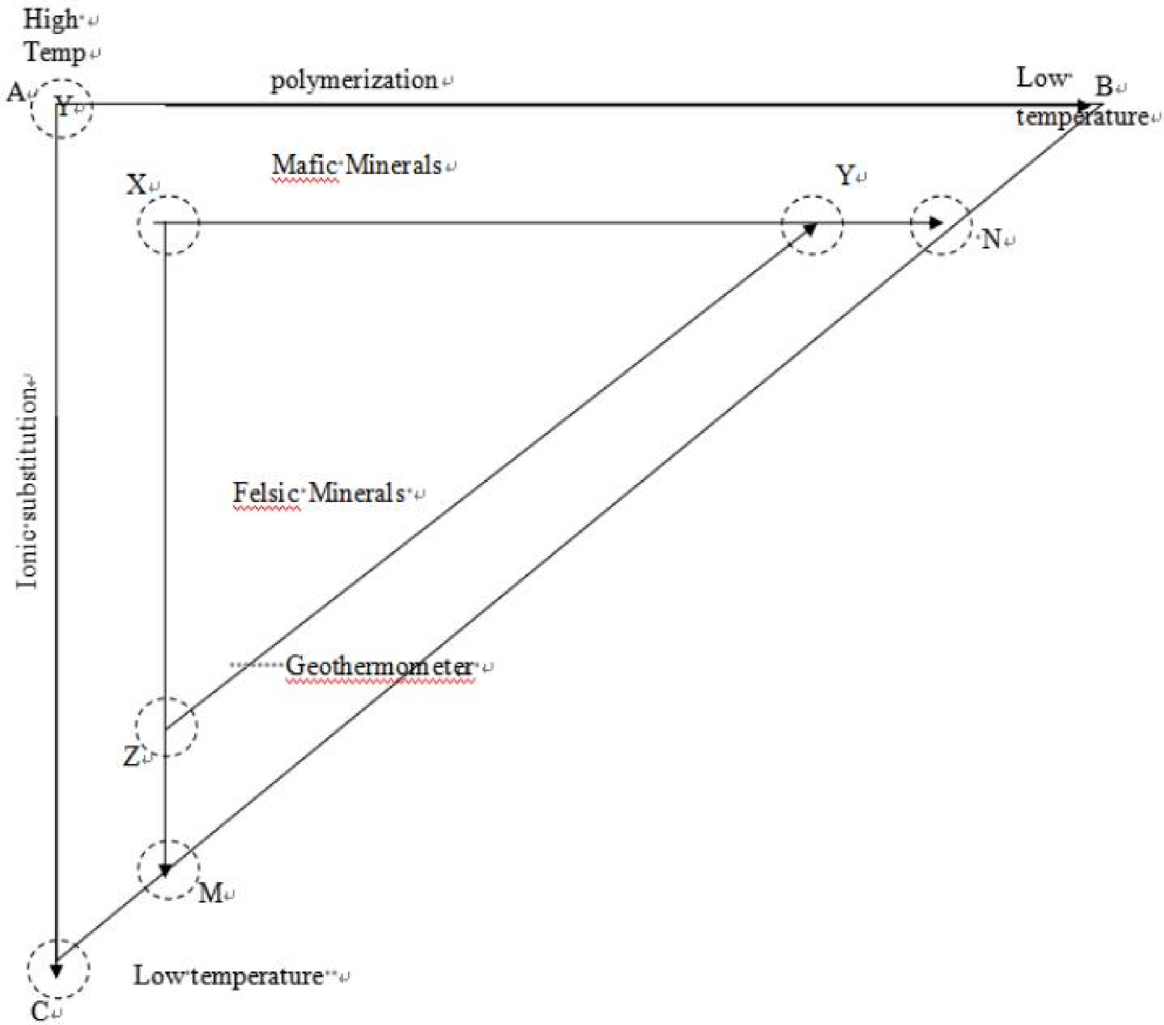

Figure 1. Schematic diagram showing the position of felsic and mafic minerals in the rock during crystallization of magma

X, Y, Z = Felsic minerals

XZ = Plagioclase series
YZ = Alkaline series=geological thermometer
A, B, C = Mafic minerals
AC = Ionic substitution
AB = Fractural crystallization

3.3 Mathematical Method

The mathematical methods include

1. Matrix
2. Set Notation
3. Geometry

In analogy, in modeling of rocks, the mathematical expression adopted in this research are $\sum_{x=0}^{\infty} (X_n Z)$and $\sum_{x=0}^{\infty} (X_{n-X} Y_X Z)$ where X and Y are the cations that balance the radical or anion Z, during ionic substitution and the subscript n, depends on the charge balance of the radical Z, by cations X and Y, as the numerical values of x range from zero (0) to infinity (∞) on the condition that $n \geq x$

4. Results and Discussion

The rock samples that were collected from Kassa Basalt were observed under thin section and presented in the Table 1, below.

Table 1. Majorn Oxides Compositions of KASSA and MIANGO basalts

	Mineral		Percentage composition of normative minerals of selected Basalts					Percentage Composition of normative minerals as it is observed from Thin Section by counting under Microscope and chemical analysis.									
			Olivine basalt Magma Type	**Tholeiitic Magma Type**	**Tristan Olivine basalt, P.E.Baker et al, (1964).**	**Hualalai Alkali basal, Yolde and Tilley (1962).**	**Kilauea Tholeiite, Yoder and Tilley, (1962)**	**KASSA**					**MIANGO**				
S/N	***Geologic Name***	***Genetic Type***	***1***	***2***	***3***	***4***	***5***	***KIA1***	***KIA2***	***KIA3***	***KIA4***	***KIA5***	***MIA1***	***MIA2***	*MIA3*	*MIA4*	*MIA5*
1	Quartz			0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	Olivine			5	10.6	18.5	0.0	8.0	0.0	0.0	14.0	35	35	20.0	17	0.0	0.0
2		Augite			29.1	20.9	22.0	17.0	1.0	4.0	11.0	10.0	10.0	0.0	0.0	0.0	0.0
3	Pyroxene	Clino-pyroxene		51	0.0	0.0	22.4	0.0	0.0	20	0.0	0.0	0.0	10.0	10	45.0	45.0
4		Labradorite		30	0.0	0.0	0.0	21.0	0.0	25	22.5	12.5	20	0.0	0	15’0	12.5
		Bytownite			0.0	0.0	0.0	26	1.0	26	27.5	17.5	25	0.0	5	20.0	17.5
	Plagioclase Feldspar	Anorthite			20.4	23.6	26.1	0.0	0,0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		Albite			3.2	20,0	21.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	Alkali Feldspar	Orthoclase			12.2	5.3	2.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6		Magnetite		9	4.1	4.5	1.9	7.0		25.0	25.0	25.0	10.0	0.0	0.0	20.0	20
7	Opaque	Ilmenite			7.9	4.3	3.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	Feldspathoid	Nepheline			11.1	2.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	Rest			5	0.0	0..1	0.1	+21	+97	0.0	0.0	0.0	0.0	+70.0	+68	0.0	0.0
9	Apatite				1.4	0.7	0.3	0.0	0,0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

4.1 Mathematical Expression

At rhyolitic and basaltic melt temperatures, magmas behave like weak electrolyte, especially on a microscopic scale and are applicable to principle of electroneutrality. At an igneous temperature, there is a mobility of ions or atoms in the magma solution, since it behaves like a weak electrolyte under electrolytic condition according to Faraday, within the magma, such that under control condition of temperature, ions or atoms of the same charge or similar size must find themselves and occupy a lattice position to form crystals, which set in matrix, to form rock of a particular composition.

Since magnesium crystalizes at an igneous temperature, with respect to iron under electrolytic condition, there is always a solid solution between magnesium and iron and can be mathematically expressed in the following equations below

These derivations are based on charge balance of the constituent species, according to Denbigh, which states that, the principle of electroneutrality requires that the ionic species in an electrolyte solution remain charge balanced on a microscopic scale [9]. The requirement of electroneutrality arises from the large amount of energy required to separate oppositely charged particles by any significant distanct against coulombic forces. The electroneutrality condition of charge balance among the species in solution, as shown in equation (1) is mathematically expressed bellow.

$$\sum Z_i M_i + \sum Z_j M_j = 0 \qquad (1)$$

In whicch Z_i and Z_j are the ionic charges on basis and secondary species. It is useful to note, however, that electroneutrality as assured when the components in the basis are charge balanced.

The chemical expressions of olivine are $(MgFe)_2SiO_4$ or$(Mg_{xo}Fe_{1-xo})SiO_4$ as shown in Apendix A in A1 where x_0 denotes the fraction of Mg atoms in olivine. The variable xo, then is equivalent to variable y_1 which is measured in %. When xo= 1, (or y = 100), the formula becomes Mg_2SiO_4 for forsterite. Orthopyroxene has chemical composition $(Mg_{xp}Fe_{1-xp})$ SiO_3as shown Apendix A in A2. Where xp = 1, (y2 = 100), we have the end member enstatite; when xp = 0, the pyroxene is called ferrosilite. The functionalrelationship between xo and xp was studied by several authors include Ramberg and De Vore (1951).

With respect to Denbigh equation above, which states that the principle of with respect to Denbigh equation above, which states that the principle of electroneutrality requires that the ionic species in an electrolyte solution remains charge balanced on a microscope scale, the derivation of mafic, felsic, and oxide minerals depend on the ionic species in the molten magma during cooling and are derived in the following ways;

4.1.1 Mathematical Derivation for Mafic Olivine

SiO_4: Is the basic building block of all silicate minerals.

- SiO_4 tetrahedral: Olivine
- $SiO_4 + SiO_2 \rightarrow Si_2O_6$: Pyroxene

Total anion for olivine

$(Si^{+4} + O^{2-}) = 0$

Since the algebraic sum of constituent elements in a compound is equal to zero

$(4 \times 1)-(4 \times 2) = 0$

$4-8 = 0$

SiO_4^{-4} = Olivine radical

$SiO_4 = -4$

Therefore, cation is needed to balance this olivine radical, so that the total sum would be zero.

Let the cation be X and n, number of ionic spacies in the magma, so that reaction in molten magma is represented by the formula

$[n(X)^+(Z)^-]$, which is the parent magma.

Therefore, $[n(X)^+(Z)^-] = 0$,so that,

$$[n(X)^+(Z)^-] = (Xn)Z \qquad (2)$$

Where Z is the radical and

$$n = \frac{Z^-}{X^+} \text{ (ionic spacies)}$$

For divalent metal and tetrahedral radical,

$X^+ = +2$

$Z^- = -4$ e.g. (olivine radical)

$n = \frac{4}{2}$, so that

$(Xn)Z = (\frac{4}{2}X)Z$

$= 2X_2Z_2$

$(2X)Z$, where $2X = X_2$

$$(XnZ) = X_2Z \qquad (3)$$

During the cause chemical reaction with falling temperature of magma, cation would be substituted by more or less electropositive cation so that equation (3) becomes

$(X_n\text{-}xYx)Z = (X_2\text{-}xYx)Z$ (4)

Where, Y is the recipient cation, Z is the anion and X is the cation.

Putting, $X = Mg$

$Y = Fe$

$Z = SiO_4$, we re-write

Equation (3) as,

$(X_2\text{-}xYx)Z = Mg_{2\text{-}X}Fe_XSiO_4$ (5)

Equation (5) is a solid solution, where x is an integer ranges from 0 to 2 in olivine crystals. At x equals to zero, Forsterite (Fo) crystallizes because of small size of magnesium (Goldschmidt 1930). At x equals to 2, forsterite disappears and 100% of pure Fayalite crystallizes with chemical formula of $Fe_2SiO_{4(s)}$.

4.1.2 Mathematical Derivation for Mafic Pyroxene

Pyroxene is a pseudomorph of olivine. This is because the SiO_4 terahedral of olivine during cooling of magma reacts in unison with silica comprising the composition of remaining fluid form pseudomorphic olivine called pyroxene.

$Mg_2SiO_4 + SiO_2 \rightarrow MgSi_2O_6$

$\downarrow \qquad \downarrow$

$SiO_4 + SiO_2 \rightarrow SiO_6$: Pyroxene radical

Unison

Total anion for pyroxene

$[(Si)^{4+} (30)^{-2}]$

$[(Si)^{4+} (30)^{-2}] = 4 + (-6) = 4\text{-}6$

$Si_2O_6 = -2$

$Si_2O_6 =-2$

To balance the anion during reaction, cation must be added.

From equation 2,

$[n(X)^+(Z)^-] = (nX)Z$ and

$n = \frac{z^-}{x^+}$

$n = \frac{2}{2}$

$\therefore (XnZ) = (\frac{2}{2}X)Z$

$= X_2Z_2$

$(XnZ) = X_2Z_2$

$(XnZ) = X_2Z_2$ (6)

If $Z = Si_2O_6$

$(XnZ) = XSiO_3$ (7)

During crystallization, there is always a solid-solution, in which there is exchange of ions between the cations, so that equation (7) becomes

$(X_{n\text{-}X}Y_X)Z = (X_{1\text{-}}xYx)Z$ (8)

Where Yx is the recipient cation.

Putting, $X = Mg$

$Y = Fe$

$Z = SiO_3$

Equation 7 becomes,

$(X_{1\text{-}X}Y_X)Z = (Mg\text{-}xFe_X)SiO_3$ (9)

Equation (9) is a solid solution, where x range from 0 to 1 in pyroxene. At x equals to zero, Enstatite (En) crystallizes, and at x equals to 1, Enstatite disappears and 100% of pure Ferrosilite crystallizes with chemical formula of $Fe_2Si_2O_6$ as shown in Apendix A in A2.

Mathematical Derivation for Mafic Amphiboles

In amphiboles, the Si_2O_6 group in pyroxene links to cation is doubled and this is shown equation (10) below.

$Si_2O_{6(s)} + Si_2O_{6(s)} \rightarrow Si_4O_{12(s)}$ (10)

In the presence of the adequate water, which is represented in the form of hydroxyl group, at least one atom of oxygen is being replaced by one atom.

$$H_2O_{(aq)} \rightleftharpoons \underbrace{OH^-_{(aq)} + H^+_{(aq)}}_{\text{Ionization}}$$

$Si_4O_{12(s)} + OH^-_{(aq)} \rightarrow Si_4O_{11}OH$ (11)

The total anion for Amphibole radical (Z)

$Z = Si_4O_{11}(OH)$

$= (4Si)^{4+} + 11(O)^{2-} + 1\ (OH)^{-1}$

$= (4 \times 4) + 11\ (2\ x\text{-}11) + (1x\ \text{-}1)$

$16 + (-22) + (-1)$

$16\text{-}22\text{-}1$

$-\ \ 6\text{-}1$

$=-7$

$Z = -7$

Since, Z is not equal to zero, then the crystal not in equilibrium with the magma and cation is needed to balance the anion so that the algebraic sum would be equal to zero.

From equation 2.

$[(nX)^+(Z)^-] = (XnZ)$

$n = {}^7/_2$

$(XnZ) = ({}^7/_2X)Z$

$Z = -7$

$X = 2$

$(XnZ) = X_7Z_2$ (12)

Equation (12) is a double-chained pyroxene because of twice the number of Z molecule.

During the cause of chemical reaction with falling temperature, the cation could be substituted by more or less electro-positive cation so that equation (12) becomes.

$(X_{n\text{-}X}Y_X)Z_2 = (X_{7\text{-}X}Y_X)Z_2$ (13)

If $X = Mg$

$Y = Fe$

$Z = Si_4O_{11}(OH)$

Then;

$(X_{7\text{-}X}Y_X)Z_2 = [(Mg_{7\text{-}X}Fe_X)(Si_8O_{22}(OH)_2]$ (14)

Equation (14) is a solid-solution, when x ranges from 0 to 7. At x equals to zero, Kupfferite (Ku) crystallizes, because of small size of magnesium than iron, though the same charge (+2), Goldschmidt (1930). At x equals 7, Kupfferite disappears, and 100% pure Grunerite (GR) crystallizes with chemical formula of $Fe_7Si_8O_{22}(OH)_2$ as shown in Apendix A in A3.

Under metamorphism, at x equals to zero, from equation (14), there is a replacement of atoms of x by another atoms of similar charges to form Tremolite as shown in equation (15) below.

$$(X_{7-X}Y_X)\,Z_2 = [(X_5M_2)\text{-}xYx]\,Z_2 \qquad (15)$$

$$= (X_5M_2)Z_2 \qquad (16)$$

Where X = Mg

M = Ca

$Z = Si_4O_{11}(OH)$

Y = Fe

$$(X_{7-X}Y_X)Z_2 = Mg_5Ca_2Si_8O_{22}(OH)_2 \qquad (17)$$

Tremolite

At x equals to 7, there is a complete solid solution, such that there is a replacement of atoms of Y by another atoms of similar charges to form Arctinolite as shown in equation (18) below.

$$(X_{7-X}Y_X)Z_2 = (Y_5M_2)Z_2 \qquad (18)$$

Actinolite

$$= (Fe_5Ca_2)Si_8O_{22}(OH)_2 \qquad (19)$$

(Actinolite)

4.1.3 Mathematical Derivation for Mafic Amphiboles with Respect to Couple Substitution

There is also a couple substitution among the amphiboles, as accordingly by Goldschmidt norms, that element can be substituted for another different charge but only when there is a couple substitution so that electrically neutrality in crystal lattice is maintained. In order to maintain electrical neutrality, different loops are being resolved such as calcium and silicon position as shown in the equation (20) below.

$$(X_5A_2)Z = (2A^{+2})\quad X_5\quad (Q_8^{+4})O_{22}(OH)_2 \qquad (20)$$

A = Ca (loop1) Q = Si (loop2)

$$(X_5A_2)Z= (A_2)X_5(Q_8)i \qquad (21)$$

$i = O_{22}(OH)_2$

LOOP (1)

Calcium position;

In calcium position, let, A = Ca

So that, $A_2 = Ca_2$.

When there is change in condition such temperature, sodium is precipitated and added to calcium position so that,

$Ca_2Na = A_2B$, where B = Na

LOOP (2)

Silicon Position

In silicon position, let M = Si

$M_8 = Si_8$

Under the same condition, aluminum proxies for 2 molecules of silicon, so that,

$Si_6Al_2 = Q_6N_2$

Q = Si

N = Al

Therefore, $(X_5A_2)Z_2 = (Fe_5Ca_2)Si_8O_{22}(OH)_2$ becomes,

$$(A_2B)(X_5)(Q_6.N_2)i = (Ca_2Na)(Mg_5)Si_6Al_2O_{22}(OH)_2 \quad (22)$$

$$(X_5A_2)Z_{(aq)} + B_2O_{(s)} + FeO + N_2O_{3(s)} \longrightarrow (A_2B)(X_4Fe)(Q_6\,N_2)O_{22}(OH) + BON_2Q_2O_3 + \tfrac{1}{2}O_2 + XO \quad (23)$$

pargasite

A = Ca

B = Na

X = Mg

Q = Si

N = Al

In the presence of aluminium and iron oxide, with following temperature, tremolite reforms into pargasite $(A_2B)(X_4Fe)(Q_6.N_2)O_{22}(OH)_2$ and there a substitution of magnesium (X) by iron (Y) and silicon (Q) by aluminium (N) as sodium (B) is being added to calcium (A) from equation (23).

Pargasite transformed into ferro-hastingsite by complete substitution of magnesium (X) by iron ($^-$Y) in solid solution as shown in equation (24).

$$(A_2B)(X_4Y)(Q_6.N_2)O_{22}(OH)_2 + YO_{(s)} \longrightarrow (A_2B)(Y_4Y)(Q_6.N_2)O_{22}(OH)_{2(s)} + \tfrac{1}{2}O_2 \quad (24)$$

Y = Fe

4.1.4 Mathematical Derivation for Arfvedsonite

Sodium replaces calcium in some proportion to form arfvedsonite and it is modeled below.

From loop (1) in $(A_2B)(X_4Y)(Q_6.N_2)O_{22}(OH)_2$.

Loop (1) = A_2B

If the composition of the element is homogeneous then,

B = A, so that,

$A_2B = A_3$

Such that, A_3 = Calcium position

In the calcium position, there is a substitution of calcium by sodium as temperature drops, during solid solution. Therefore, let the sodium atom be B, and its molecular fraction be x so that,

$B = B_x$

Then, in calcium position, we have;

$A_{3-X}B_X = 0$

$$A_2B = A_{3-X}B_X \quad (25)$$

From the equation $(A_2B)(X_4Y)(Q_6N_2)O_{22}(OH)_2$.

Loop (2) = Q_6N_2

If the composition of the loop (2) is a homogeneous element then,

Q = N, so that,

$Q_6N_2 = Q_6Q_2$

$= Q_8$

Therefore, Q_8 = silicon position

In the silicon position, there is substitution of 2 moles of silicon by 2 moles of aluminium as temperature drops,

Therefore let the aluminium atom be N, and its molecular fraction be x, so that;

$N = N_Y$

Then in silicon position, we have;

$Q_{8-Y}N_Y = 0$

$$Q_8 = Q_{8-Y}N_Y \quad (26)$$

Loop (1) = $A_{3-X}B_{X,}$ so that

$A_{3\text{-}X}B_X$ = calcium position

Loop (2) = $Q_{8\text{-}Y}N_Y$, so that;

$Q_{8\text{-}Y}N_Y$ = silicon positon

Put the entities $A_{3\text{-}X}B_X$ and $Q_{8\text{-}Y}N_Y$ in the equation

$(A_2B)(X_5)(Q_6N_2)O_{22}(OH)_2 = 0$

$A_2B = A_{3\text{-}X}B_X$

$Q_6N_2 = Q_{8\text{-}Y}N_Y$, then

$$(A_2B)(X_5)(Q_6N_2)O_{22}(OH)_2 = (A_3\text{-}xB_X)(X_4Y)(M_{8\text{-}Y}N_Y)O_{22}(OH)_2 \quad (27)$$

$$(A_3\text{-}xB_X)(X,Y)_5(M_{8\text{-}Y}N_Y)O_{22}(OH)_2 = (Na_5Ca)(Mg,Fe,Al)_{10}(Si_{15}Al)O_{44}(OH)_4 \quad (28)$$

And this represents a couple substitution, as stated by Godlschmidt, where $(Na_5Ca)(Mg,Fe,Al)_{10}(Si_{15}Al)O_{44}(OH)_4$ is the chemical formula for Arfvedsonite.

4.1.5 Mathematical Derivation for Mafic Micas

The crystal of amphibole interacts with the magma, so that the Si_4O_{12} of amphibole links to cation is being hydrolysed in the presence of excess water in the magma shown in equation (29).

$$Si_4O_{12(s)} + 20H_{(aq)} \longrightarrow Si_4O_{10}(OH)_2(aq) \quad (29)$$

In equation (29), two atoms of oxygen are being replaced by two atoms of hydroxide atoms in $Si_4O_{12(s)}$ to form $Si_4O_{10}(OH)_{2(aq)}$. Also, hydrolysed amphibole links to cation is doubled, and the radical becomes $Si_8\ O_{20}\ (OH)_{4(aq)}$.

Total radical for mica (Z)

$Z = Si_4O_{10}(OH)_2 = 0$

$Z = [4_{Si}{}^{+4}] + 10\ [0^{2-}] + 2\ (OH^-) = 0$

$[4x4] + [10x\text{-}2]\ ]2x\ \text{-}1] = 0$

$16\ \ \text{-}\ 20\text{-}2 = 0$

$\text{-}4\text{-}2 = 0$

$\text{-}6 \neq 0$

$Z = \text{-}6$

$Z \neq 0$

Cation is needed to balance the radical (Z) so that the algebraic sum would be zero.

$[(nX^+)(Z^-)] = (XnZ)$ and

$z = \text{-}6$, and $X = +2$

$n = {}^6/_2$

$(XnZ) = {}^6/_2(XZ)$

$= 3X_2Z_2$

$3XZ = X_3Z$

$$(XnZ) = X_3Z \quad (30)$$

For double chained amphibole, we have

$(XnZ) = (X_3Z)_2$

$$= X_6Z_2 \quad (31)$$

During the cause of chemical reaction with falling temperature, the cation would be substituted by more or less electropositive cation so that equation (31) becomes

$$(XnZ) = (Y_{6\text{-}X}Y_X)Z_2 \quad (32)$$

Putting X = Mg

Y = Fe

$Z = Si_4\ O_{10}\ (OH)_2$

Equation (30) becomes

$$(Y_{6-X}Y_X)Z_2 = [Mg_{6-X}Fe_X][Si_8O_{20}(OH)_4] \quad (33)$$

Equation (48) is a solid solution, when x ranges from 0 to 6. At x equals to zero, phlogopite (Ph) crystallizes, and at x equals to Biotite disappears and 100% pure Lepidomelane crystallizes with a chemical formula of $K_2Fe_6Al_2O_{20}(OH)_4$as shown in Apendix A in A4. The above forms a biotite series.

At the last temperature, where biotite is formed, aluminum substitutes for iron in all proportion to form the white mica with increasing content of alkaline metal (K, Na) called muscovite mica. In the presence of Lithium at a low temperature Lepidolite and Zinnwaldite appear. In the case where x = 6, from equation (33).

$$[X_{6-X}Y_X](Z_2) = Y_6\ Z_2 \quad (34)$$

$$\text{If } Y_6Z_2 = (P_2\ Q_4) \quad (35)$$

$$Y_6Z_2 = [(P_2Q_4)Z_2] \quad (36)$$

Where P is the alkali metal (K, Na) and Q is the proxyl aluminium (Al), then

$$(P_2Q_4)Z_2 = K_2Al_4Si_8O_{20}(OH)_4 \quad (37)$$

Muscovite (Mv)

If sodium is exchanged for potassium in equation (37), then it is called paragonite as shown in the equation (38) below,

$$(P_2Q_4)Z_2 = Na_4Al_4Si_8O_{20}(OH)_4 \quad (38)$$

Paragonite

Therefore, equation (38) is the paragonite.

4.1.6 Mathematical Derivation for Felsic Minerals

- For Nepheline silicate chaine (Radical).

$$-Si_2O_4 = 0$$

$$[2Si]^{4+} + 4\ [0]^{2-} = 0$$

$$[(2 \times 4) + 4(-2)] = 0$$

$$8-8 = 0$$

- Nepheline radical, equilibrium with Magma at high temperature.
- Because of concentration of aluminum in the magma, aluminum replaces one atom of silicon in the solid-solution, which left a surplus negative charge as shown below.

  The total nepheline radical;

$$AlS_iO_4 = 0$$

$$[Al]^{3+}{}_{+}\ [Si]^{4+} + 4\ (O)^{2-} = 0$$

$$3 + 4-8 = 0$$

$$7-8 = 0$$

$$-1 \neq 0$$

- As temperature drops, nepheline not in equilibrium with the magma
- Cation is needed to balance this surplus ion, so that the algebraic sum would be zero.

$$\text{If } AlSiO_4 = -1$$

$$Z = -1$$

Then for divalent metal

$$X = +2$$

From equation 2

$$[(nX^+)(Z^-)] = (XnZ)$$

$$n = ½$$

$$(XnZ) = ½(XZ)$$

$$= XZ_2$$

$$(XnZ) = XZ_2 \quad (39)$$

For double chain radical (Z),

$$(XnZ) = XZ_2 \quad (40)$$

If there is a couple substation among the cation and radical (Z), then equation 40 becomes $(X_{1-X}Y_X)Z_2$

$$(X_{n-X}Y_X)Z_2 = (X_{1-X}Y_X)Z_2 \quad (41)$$

Where Y_X is the recipient cation and where $Z_2 = Al_2Si_2O_8$, such that for

Silicon position

Al_2Si_2 = Si-position

$Al = Si$

$Al_2Si_2 = Si_2Si_2$

$= Si_4$

$Si = A$

$Si_4 = A_4$

Silicon position = A_4

- Let the number of aluminum atom B, be Y
- Aluminum replaces one atom of silicon. In silicon position as shown below

$B = By$

$A_4 = (A_{4-Y})(B_Y)$

$$A_4B = (A_{4-Y})B_Y \quad (42)$$

Silicon position = $A_{4-Y}B_Y$

$\therefore Z_2 = Al_2Si_2O_8$

$$Z_2 = A_{4-Y}B_YO_8 \quad (43)$$

$\therefore$ Put the value of Z_2 in equation (43).

$XZ_2 = (X_{1-X}Y_X)Z_2$

$(X_{1-X}Y_X)Z_2 = (X_{1-X}Y_X)(A_{4-Y}\ B_Y)O_8.$

$$(X_{1-X}Y_X)Z_2 = (X_{1-X}Y_X)(A_{4-Y}By)O_8 \quad (44)$$

Equation (44) is a solid solution which undergoes couple substitution when x ranges from 0 to 1, and y ranges from 1 to 2.

At x equals to 0 and y equals to 1, sodic feldspars are formed. At x equals to 1, and y equals to 2, calcic minerals are formed at high temperature.

As the magma differentiated from basaltic magma to the rhyolite magma, temperature decreases as the viscosity increases with the depletion of Ca, Mg, and Fe and enriched in the variable concentration of Na, K, Si, and Al as shown in the following equations bellow. To prove that calcium element is completely depleted from the melt and sodium and potassium are variable with constant silicon and aluminum within ΔXYZ, as temperature drops during crystallization of magma.

4.1.7 Mathematical Expression of Perthite-Antiperthite Exolution

From ΔXYZ, in figure 2 and 7;

X = Ca

Y = K

Z = Na, such that

XYZ = felsic mineral.

If XZ = Ca, K

XZ = Ca, Na

YZ =?

Let, YZ = M

Using Pythagoras theory, we have,

$$|XY|^2 + |XZ|^2 = |M|^2 \quad (45)$$

As temperature changes with respect to composition,

$$\Delta y/\Delta x = 2XY + 2XZ,$$

As [Δy/Δx] limiting to zero (0), as temperature drops,

$2XY + 2XZ = 0$

$2X(Y+Z) = 0$

$$X(Y+Z) = 0 \quad (46)$$

$X = 0$

$Ca = 0$

Equation (46) shows, that calcium is depleted from the magma as temperature drops,

Therefore, $Y + Z = 0$ (high temperature) (47)

This means that Z and Y exist at relative high temperature (rhyolitic temperature) that produces mixed alkali feldspar called Perthite-Antiperthite mixture.

But, at a low temperature;

$Y = -Z$ (low temperature) (48)

$+K = -Na$ (low temperature Perthite) (49)

This means that potassium ion dominates sodium ion, and there are patches of sodium m within potassic dominated rock, called Perthite.

Also, contrary to equations above,

$-Y = +Z$ (low temperature) (50)

$-k = + Na$ (low temperature Antiperthite) (51)

This means that, sodium ion dominates potassium ion, which indicates that, there are patches of potassium ion within sodic dominated rock, and is called Antiperthite.

- Note that the positive and the negative signs in the above equations indicate the dominance of one feldspar over the other.

5. Application of Mathematics to Rock Forming Minerals Using Certain Aceptable Mathematical Parametres

The mathematical parametres include;

- Matrix
- Set Notation
- Coordinate Geometry

These parametres are used to analyse rock forming minerals from the start of crystallization to the end of crystallization.

5.1 Matrices Method

In mathematics,a matrix (plural: matrices) is a rectangular array of numbers, symbols, or expressions, arranged in rows and columns. The numbers are called the elements, or entries, of the matrix. Matrices have wide applications in engineering, physics, economics, and statistics as well as in various branches of mathematics.

Given that,

Basic Plagioclase = BaP
Acidic Plagioclase = AcP
Basic = Ba
Acidic = Ac

Considering equation (134) and (136)

Such that,

$BaP + AcP = F$ (52)

$Ba + Ac = M$ (53)

If $[(X_{1-X}Y_X)(A_{4-Y}B_Y)]O_8$ = Felsic and

$(X_{2-X}Y_X)Z$ = Mafic

Then,

$[(X_{1-X}Y_X)(A_{4-Y}B_Y)]O_8 = 0$ (54)

$(X_{2-X}Y_X)Z = 0$ (55)

In this case factor theorem is used, to evaluate the simultaneous equations above. In algebra, the factor theorem is a theorem linking factors and zeros of a polynomial. It is a special case of the polynomial remainder theorem

The factor theorem states that a polynomial has a factor if and only if (i.e. is a root) When $x = 0$ and $y = 2$, then,

$(X_{1-X}Y_X)(A_{4-Y}B_Y)O_8$ becomes

$(X_1B_2A_2)O_8 = 0$ (56)

$X_2Z = 0$ (57)

When $x = 1$ and $y = 1$, then,

$(X_{1-X}Y_X)(A_{4-Y}B_Y)O_8$ becomes

$(Y_1BA_3)O_8 = 0$ (58)

$Y_1Z = 0$ (59)

Where, X_1 = Mg, Ca for Basic

Y_1 = Fe, Na for Acidic

Z = SiO_4

From equation 5

$(X_1B_2A_2)O_8 = 0$

$CaAl_2Si_2O_8 = 0$, for plagioclase

$X_2Z = 0$

$Mg_2SiO_4 = 0$, for Basic

From equation (7)

$(Y_1BA_3)O_8 = 0$

$NaAlSi_3O_8 = 0$, for Alkali

From equation 8

$Y_2Z = 0$

$Fe_2SiO_4 = 0$, for Acidic

$CaAl_2Si_2O_8+NaAlSi_3O_8 = 0$ (60)

$Mg_2SiO_4+Fe_2SiO_4 = 0$ (61)

$CaAl_2Si_2O_8 + NaAlSi_3O_8 = 0$

(0, 2) : (1, 1)

$Mg_2SiO_4 + Fe_2SiO_4 = 0$

(0, 0) : (2, 0)

Then in matrix form, we have

$$\begin{pmatrix} Pl & Al \\ Ba & Ac \end{pmatrix} = \begin{pmatrix} felsic \\ mafic \end{pmatrix} \quad (62)$$

$$\begin{pmatrix} Anorthite \\ forsterite \end{pmatrix} + \begin{pmatrix} Albete \\ Fayalite \end{pmatrix} = \begin{pmatrix} Andesine \\ Monticellite \end{pmatrix} \quad (66)$$

$$\begin{pmatrix} X_1 & B_2 & A_2 \\ & X_2Z & \end{pmatrix} + \begin{pmatrix} Y_1 & BA_3 \\ & Y_2Z \end{pmatrix} = \begin{pmatrix} X_i & Y_1 & B_3A_5 \\ X_2 & Y_2 & Z \end{pmatrix} \quad (67)$$

$$\begin{pmatrix} C_a & Al_2 & S_{i2}0_8 \\ Mg_2 & S_1 & 0_4 \end{pmatrix} + \begin{pmatrix} Na & Al & Si_30_8 \\ Fe_2 & Si & 0_8 \end{pmatrix} = \begin{pmatrix} Ca & Na & Al_3S_{i5}0_{16} \\ (Mg \quad Fe)_2 & S_1 & 0_4 \end{pmatrix} \quad (68)$$

$$\begin{pmatrix} 0\ , & 2 \\ 0, & 0 \end{pmatrix} + \begin{pmatrix} 1\ , & 1 \\ 2\ , & 0 \end{pmatrix} = \begin{pmatrix} 1\ , & 3 \\ 2, & 0 \end{pmatrix} \quad (69)$$

[Basalt] + [Rhyolite or Dacite] = Intermediate

If P > A, then the intermediate rock is Andesite

[Basalt] + [Dacite] = Andesite (70)

If P < A, then the intermediate rock is the Trachyte

[Basalt] + [Rhyolite] = Trachyte (71)

If A = P, then the Intermediate rock is the Hybrid Monzonite

[Basalt] + [Rhyolite] = Hybrid Monzonite (72)

From the deductions and the anology above, I can say that no rock wholly rich in magnesium is granitic in composition, because of its instability in the presence of free silica, except for the iron rich rock which is stable in the presence of free silica e.g rhyolite in the Younger Granite of Jos Plateau. Therefore, magnesium rich minerals are basic, which include basalt, dolerite, norite and gabbro. Iron rich minerals are acidic and they include granite, rhyolite, granodiorite, and dacite. The coordinate points A and B form the composition of basaltic rocks of Kassa Basaltic Volcanoe as exammined from thin section under cross-polarized light and plane-polarized light.

5.2 Set Notation Theory

Table 2. Typical Partition Coefficient of Trace Elements between Crystals and Liquid in KASSA Basalts

	Trace Elementl		Trace elements in Basaltic liquid: from J.G., Arth, 1976, Jour.Res.U.S. Geol. Surv. ,4: 41-47.						Trace elements in Basalt of KASSA from the researcher.				
		Olivine	Pyroxene	Pyroxene	Amphibole	Mica	Plagioclase	Spinel	Garnet	Converted Data in KASSA x100				
S/N		*1*	2	3	*4*	*5*	6	*7*	*8*	*KIA1*	*KIA2*	*KIA3*	*KIA4*	
1	Ni	4-10	8.3	2.5	6.0	7.6	0.05	5.0	0.5	1.54		0.17	1.68	
2	Cr	0.2	2.0	11.5	5.2	7.0	0.06	10.0	2.0	2.26		0.07	1.72	2.15
3	Co	3.9	2.4	1.0	6.5	1.1	0.05	2.0	3.2	0.46		0.35	0.49	0.50
4	Sc	0.2	1.2	2.7	3.5	3.0	0.03	2.0	3.4					
5	Sr	0.01	0.03	0.11	0.6	0.1	2.1	<0.1	<0.1	4.45		9.59	6.63	7.81
6	Ba	0.02	0.05	0.02	0.4	<0.1	0.38	<0.1	<0.1	4.23		8.77	8.89	5.11
7	Rb	0.02	0.006	0.03	0.4	2.0	0.09	<0.1	<0.1					
	Mn									1.17		1.35	1.43	1.35
	Zr									1.56		2.71	1.92	1.97
	V									1.69		1.93	1.69	1.67
	Zn									1.07		1.33	1.17	1.31

The theory of sets is an important tool in modern Mathematics. The study of sets has assumed a central role in every branch of Mathematics today. Set theory can be applied in geology to study the distribution of trace and or REE in rock after extensive geochemical analysis.

It is more precise to use set theory to probe or discriminate and delineate the distribution of trace elements in basaltic liquid, rather than using matrix method throughout the time of crystallization of magma after geochemical analysis using XRF.

Using set theory to probe or discriminate trace elements in Basaltic liquid, given the fact that D > 1, where D, = partition coefficient, then, from Table 3 above, trace elements in;

- Olivine (O_l) = Ni, Co
- Pyroxene (P_y) = Ni, Cr, Co, Sc
- Plagioclase (P_l) = Sr

Suppose that all the trace elements in olivine, pyroxene and plagioclase are included in Basaltic liquid, it is pertinent to know that, not all the trace elements in the same Basaltic liquid are included in essential olivine, pyroxene, and plagioclase. This is because; the concentration of trace elements in solid to liquid depends on its partition coefficient (D).

Given that, Basaltic liquid is a universal set of all included trace elements, $\cup$, then according to Prinz [10]

$$\cup = \{Ba, Sr, Ni, Cr, Ga. Li, V, Sc, Rb, Co, Cu\}$$

Using set notation,

If O_l, P_y, P_l, Z… X_n, are sets of included trace elements, then their union is the set of all included trace elements which belong to at least one of them and it is denoted by;

$$O_l \cup P_y \ldots\ldots \cup X_n = \cup X_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (73)$$

If the union is infinite, then, O_l, P_y, P_l, Z…X_n is given by

$$O_l \cup P_y \cup \ldots\ldots \cup X_n \cup \ldots\ldots = \cup X_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (74)$$

$$\cup X_i = \{z \in \cup\} \text{ or } \{O_l \in \cup\} \text{ or } \{P_y \in \cup\} \text{ or } \{P_l \in \cup\} \quad (75)$$

$$\cup X_i = \{Z\} \cup \{O_l \cup P_y\} \cup \{P_l\} \quad (76)$$

$$\cup X_i = \text{Basaltic liquid}$$

The intersection, ∩ of sets O_l, P_y, P_l, Z…X_n, is the sets of all included trace elements in the Basaltic liquid which belong to every one of them and is denoted by

$$O_l \cap P_y n \ldots \cap X = \cap X_i \quad (77)$$

If the sets are infinite that is,

$$O_l \cap P_y n \ldots \cap X n = \cap X_i \quad (78)$$

Let the set of included trace element in olivine, pyroxene and pyroxene are represented using set notation in Figure 3, below.

Given that,

$x \in O_l$, $x \in P_y$ and $y \in P_l$, but $z \notin \{x, y, z\}$, where x and y are set of trace elements in olivine, pyroxene and plagioclase and z is the remaining trace elements in the basaltic liquid, then

$O_l \cap P_y = \{x: x \in O_l, \text{ and } x \in P_y\}$

$= \{Ni, Co\}$

$O_l \cap P_l = \{x: x \in O_l\}$ and $\{y: y \in P_l\}$

$= Ø$

$P_y \cap P_l = \{x: x \in P_y\}$ and $\{y: y \in P_l\}$

$= Ø$

$O_l \cap P_y \cap P_l = [\{x: x \in O_l, x \in P_y\}$ and $\{y: y \in P_l\}]$

$= Ø$

$O_l \cup P_y = \{x: x \in O_l, \text{ or } x \in P_y\}$

$= \{Ni, Cr, Co, Sc,\}$

$O_l \cup P_l = \{x: x \in O_l\}$, or $\{y: y \in P_l\}$

$= \{Ni, Co, Sr\}$

$P_y \cup P_l = \{x: x \in P_y\}$ or $\{y: y \in P_l\}$

$= \{Ni, Co, Cr, Sc, Sr,\}$

$O_l \cup P_y \cup P_l = [\{x: x \in O_l, \text{ or } x \in P_y\}$ or $\{y: y \in P_l\}]$

$= \{Ni, Co, Cr, Sc, Sr,\}$

$\{Z\} \cup \{O_l \cup P_y \cup P_l\}' = [\{z: z \notin O_l, \text{ or } z \notin P_y \text{ or } z \notin P_l\}]$

$= \{Ba, Ga, Li, V, Rb, Cu\}$

From Figure 2 below, that at a given equilibrium during crystallization of magma, those trace elements within the circles are partitioned in the solid phase and are compatible trace elements, while those outside the circles are partitioned into the liquid phase and are incompatible trace elements which are ready to be partitioned, if any of the conditions is altered such as drop in tempereture. It is observed from the Figure 2 below, that olivine is a proper subset of pyroxene, with the union of plagioclase which is mutually inclusive in the Basalt.

Therefor, $O_l \subset P_y$ is read as "O_l is proper subset of P_y".

$O_l \subset P_y = O_l \cap P_y: O_l \cup P_y$

$= \{x \in O_l, \text{ and } x \in P_y\}$

$O_l \cap P_y = O_l$

$O_l \cup P_y = P_y$

$\{O_l \cap P_y\} \cap \{P_l\} = [\{x: x \in O_l, x \in P_y\}$ and $\{y: y \in P_l\}]$

$= Ø$, mutually exclusive.

$\{O_l \cup P_y\} \cup \{P_l\} = P_y \cup P_l$, mutually inclusive.

$\{O_l \cup P_y\} \cup \{P_l\} = [\{x: x \in O_l, \text{ or } x \in P_y\}$ or $\{y: y \in P_l\}]$

= Olivine Basalt

$P_y \cup P_l = [\{x: x \in P_y\}$ or $\{y: y \in P_l\}]$

= Tholeiite Basalt

Olivine Basalt ⟶Tholeiite Basalt

Undersaturation ⟶Oversaturation of Olivine.

Undersaturation of olivine called “Olivine Basalt” to oversaturation of olivine called “Tholeiite Basalt”, means that there is enough silica, according to Kennedy (1933) and included trace elements to convert all of the olivine to pyroxene. Therefore, all the trace elements in olivine are included in pyroxene, but not all the trace elements in pyroxene are included in olivine using set notation and Venn diagram as shown fFgure 4 below.

Using Vein diagrams bellow to represent the above set notations, we have;

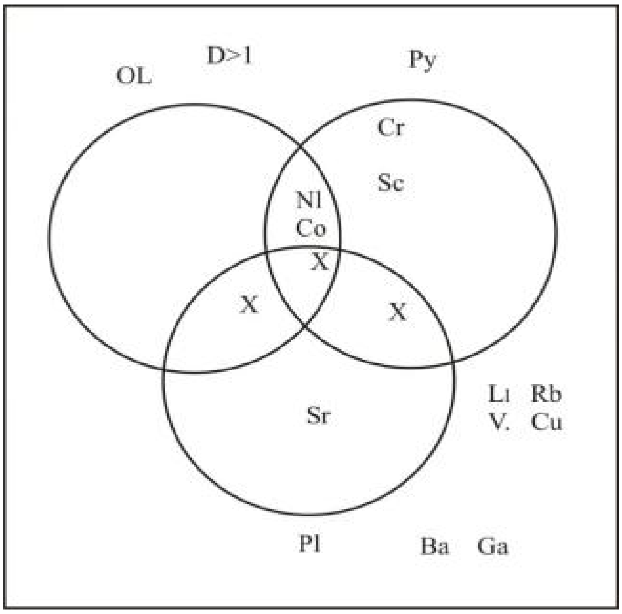

Figure 2. Vein diagram representing partition coefficient of trace element in Basalt

5.3 Analytical Geometry to Place the Minerals in Coordinates

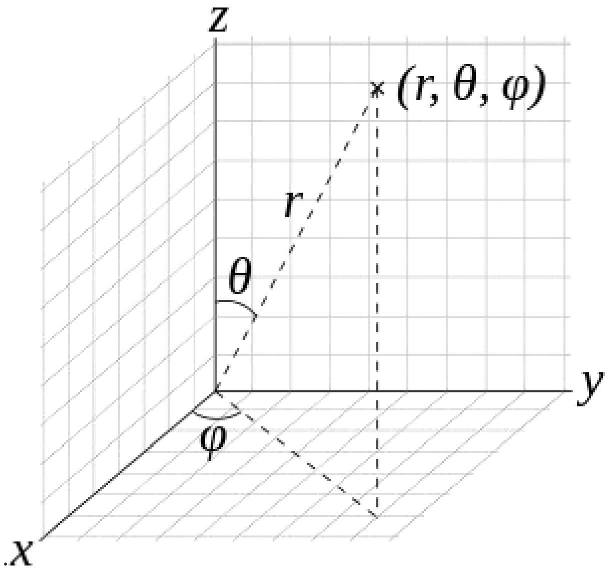

Figure 3. Coordinate System

In geometry, a Coordinate System is a system which uses one or more numbers, or coordinates, to uniquely determine the position of the points as sho\wn in Figure 6 or other geometric elements on a manifold such as Euclidean space. The coordinates are taken to be real number elementary mathematics, but may be complex numbers or elements of a more abstract system such as a commutative ring. The use of a coordinate system allows problems in geometry to be translated into problems about numbers and *vice versa*; this is the basis of analytical geometry as shown in Figure 3, above.

In analytic geometry, the plane is given a coordinate system, by which every point is a pair of real number coordinates. Similarly, Euclidean space is given coordinates where every point has three coordinates.

This coordinate geometry is used to explain the formation of rock forming minerals during crystallization of magma contrary to Bowen's reaction model. In this case, this can be translated and represented as geometric right angled-triangle in Figure 4 below where the horizontal direction represents discontinuous reaction series and the vertical direction represents Continuous reaction series.

In coordinate geometry as shown in Figure 4 below, the first member of Monticellite forms at high temperature. The early formed first member, forsterite may react with the magma by extracting silica from it in a sufficient quantity to convert into corresponding first member of Hypersthene, Enstatite. Conversion requires time for its completion, however, and in the case of lava, cooling may be so rapid that the reaction temperature is passed over too quickly for it to be effected, and some forsterite may survive, even when the magma contains, sufficient silica to convert it all to first member of Hypersthene, Enstatite, therefore, the surviving forsterite may react continuously with the magma to produce Fayalite. This process continues sequencially with Amphibole, Mica and Plagioclase.

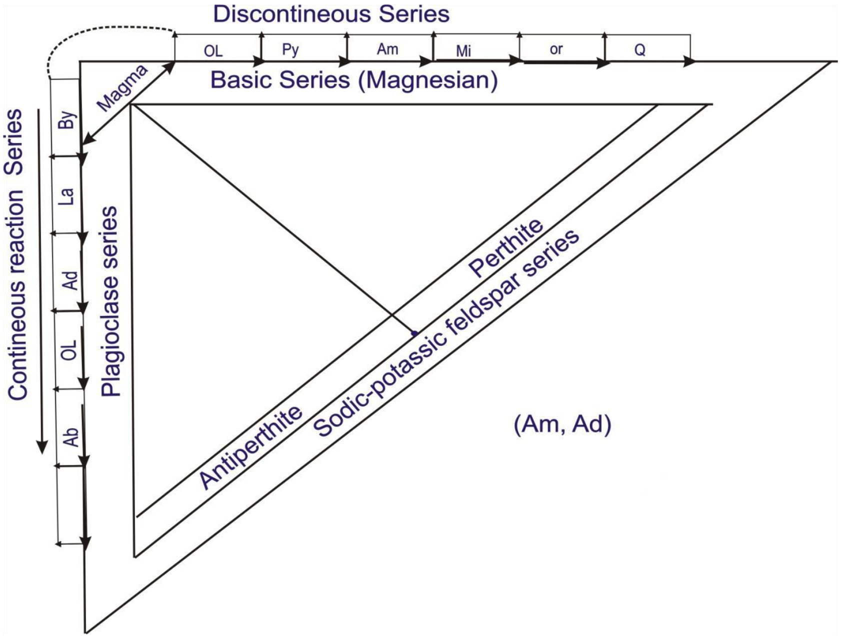

Figure 4. Geometric Right- Angled Triangle of Rock Forming Minerals during Crystallization of Magma Contrary to Bowen's reaction Model

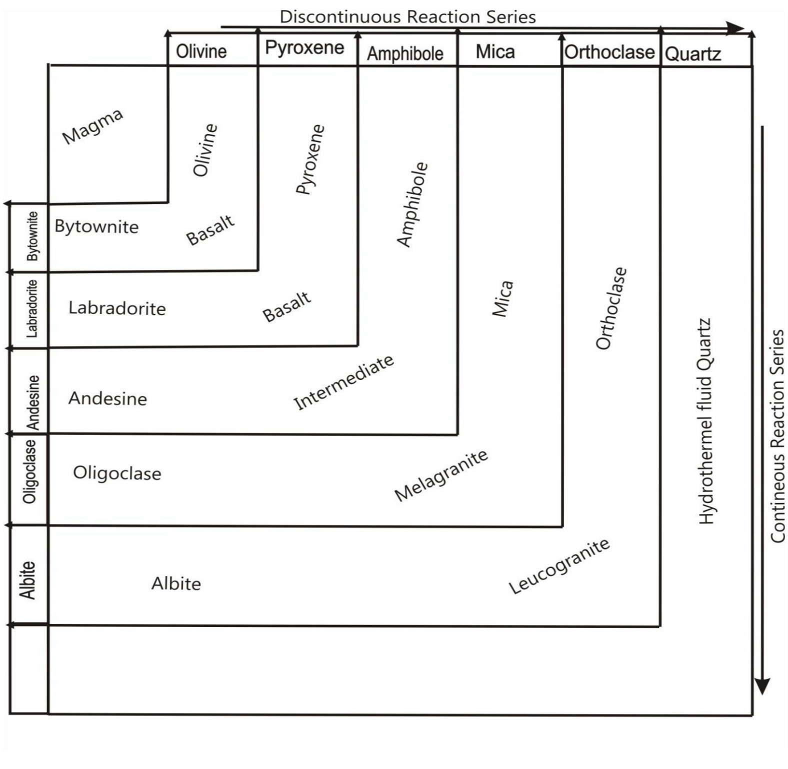

Figure 5. Geometric Relationship between continuous and Discontinuous Series of rock Forming Minerals during Crystallization of Magma Contrary to Bowen's reaction Model

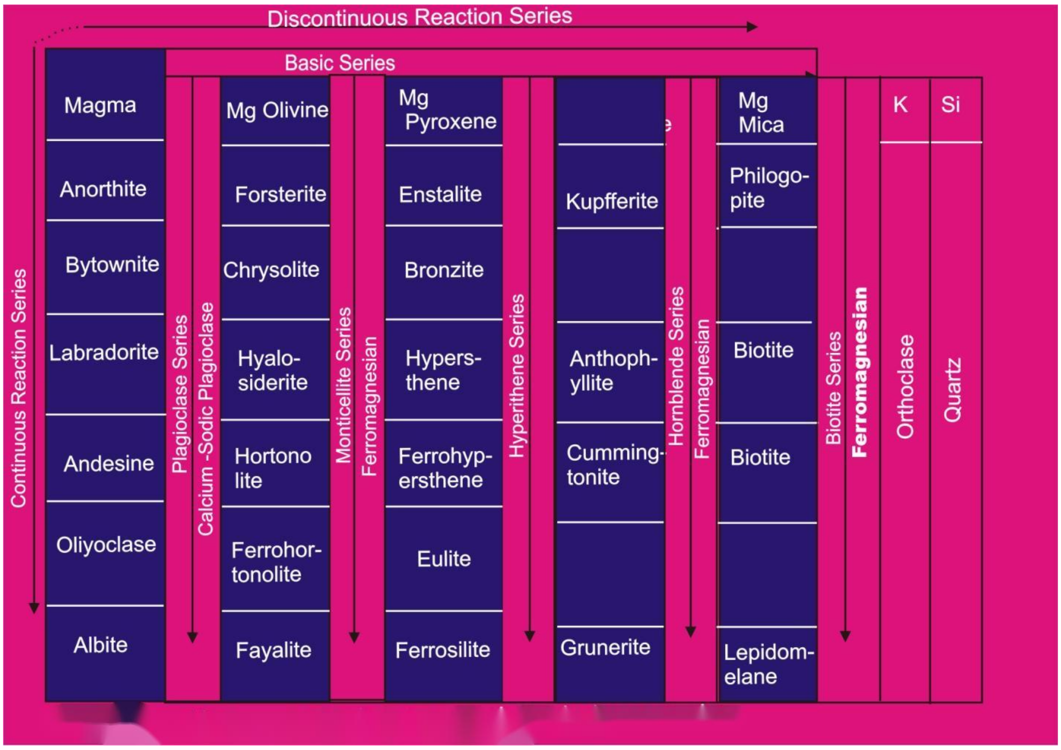

Figure 6. Geometric Sequence of rock Forming Minerals showing Ferromagnesian and Plagioclase Continuous Reaction series

The geometric relationship between the continuous reaction and discontinuous reaction series as shown in figure 8 above, contradicts the Bowen's model of reaction series as shown in figure 2 above. Figure 6 above shows that the continuous reaction series comprises both plagioclase series and ferromagnesian series, while the discontinuous reaction series comprises only the Basic series (magnesian series). This is because there is no ionic substitution of magnesium by iron at the start of crystallization in the discontinuous reaction series, but there is ionic substitution of magnesium by iron at the low temperature end of crystallization in the continuous reaction series just as ionic substitution of calcium by sodium in the plagioclase series.

If it is true that the first member of monticellite at the start of crystallization is wholly magnesium, which is unstable in the presence of free silica and is basic in classification as shown in the Appendix C in C1 and the end member at the end of crystallization is wholly iron, which is stable in the presence of free silica, and is acidic in classification as shown in the Appendix C in C1, then likewise also if it true that, the first member of plagioclase at the start of crystallization is wholly calcic which is basic in classification as shown in Appendix C in C2 and it is unstable in the presence of free silica and the end member at the end of crystallization is wholly sodic, which is stable in the presence of free silica and is acidic in classification as shown in the Appendix C in C2, then logically, it is true that calcic plagioclase and basic magnesian are basic minerals that form the Basic rock such as, Baslt, Gabbro and Dolerite as shown in the Appendix C in C4. In this case, there is insignificant or no ionic substitution that is, no solid solution between the minerals among the Bacic rocks

Similarly, it is also true logically, that sodic feldspar (alkali) and iron rich minerals are the acidic minerals that form the acidic rocks, such as Granite, Rhyolite, Granodiorite and Dacite as shown in the Appendix C in C4. In this case, there is a significant ionic substitution, that is, there is a solid solution within the same mineral, among the acidic rocks.

From the above reasons, continuous reaction series forms the following series as shown in Figure 6, above.

- Monticellite series (Ca:Mg:Fe olivine)
- Hypersthene series (Mg:Fe pyroxene)
- Hornblend series (Ca:Mg:Fe amphibole)
- Biotite series (Mg:Fe mica)
- Plagioclase series (Ca:Na feldspar)

All these minerals above are intermediate minerals that are rich in both magnesium and iron in equal proportion and are classified as ferromagnesian series in the continuous reaction series as shown in figure 8 above. Also they are also rich in calcium and sodium in plagioclase series as shown in Figure 6 above.

6. Summary, Conclusion, and Recommendations

6.1 Summary of Findings

In summary, the mathematical equations are used to place minerals in Basalts and it outlines bellow:

The starting equation for rock forming minerals during crystallization of magma is represented as

: $X_nZ = (X_{n-X}Y_Y)Z$, where x is an integer and it depends on n (Z^-/X^+), where X, Y, Z are Mg, Fe and S_iO_4

But for couple substitutions, we have

: $(X_{n-X}Y_Y)Z_n = (X_{n-X}Y_Y)(A_{4-Y}B_Y)O_8$, where X and Y represent Ca and Na

The mathematical parametres used are;

- Matrix
- Set Notation n
- Coordinate Geometry

6.2 Conclusion

In conclusion, it is shown from the Mathematical modeling, in Figure 7 below, that crystallization of the various rocks from basaltic magma takes the form of the Geometric right-angled triangle A, B, C, and X, Y, Z where ΔABC represents part of the Mafic rock, and ΔXYZ represents part of the Felsic, contrary to Bowen's reaction model.

From the Mathematical modeling, it is also concluded, that the examinations of sampled rocks collected from kassa Volcanic Basalts, under thin section, chemical analysis using ICPMS**,** and mathematical computation indicate, that minerals observed and chemical composition analyzed, fall within gabbro clan of ΔABC and ΔXYZ hybridization as shown in Figure 7 below. The gabbro clan includes basalt, dolerite, norite, scoria and gabbro. The minerals observed in these rocks under thin section are restricted to Gabbro clan and they are mainly basic, because of its instability in the presence of free silica. It is on this note, i concluded mathematically, that, Kassa volcanic Basalts are characteristics of rocks of Gabbro clan and they are olivine, alkali basalt magma type and tholeiite magma type as shown in Table 2 above.

From this research, the following conclutions are drawn mathematically. The formation of natural Basalt from Basaltic magma is a Basic rock which does not sensibly involve solid solution during crystallization. This means that, there is no ionic substitution of magnesium by iron in the Basic series. It is wholly magnesian series, no equivalent ferrous series, which indicate that, it is not actually ferromagnesian by formation. The reaction series that produces Basic series is not "a ferromagnesian" series but wholly "magnesian" series as shown in Figure 6 above.

The formation of natural Diorite from Andesite magma is an Intermediate rock which sensibly involves solid solution in the Solid solutio series. This means that, there is an ionic substitution of magnesium by iron in the Solid solution series. It is equal amount of magnesian and ferrous series, indicating that it is "ferromagnesian" series by formation as shown in Figure 6 above.

The formation of natural Granite from rhyolite magma is an Acidic rock whichis sensibly produced from solid solution. This means that, there is a partial or total substitution of magnesium by iron in the Solid solution series. It is wholly "ferrous" series, no equivalent "magnesian", indicating that it is not a "ferromagnesian" series, because of total substitution of magnesium by iron in the Solid solution series as shown in Figure 6 above. Because of this, no rock rich in magnesium is actually "Granite"

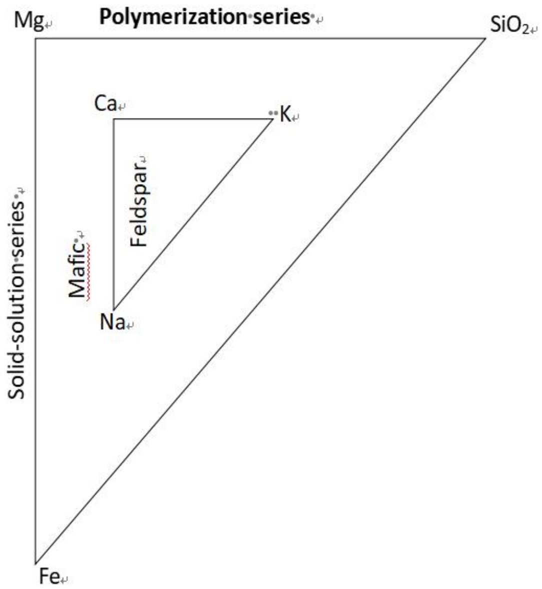

Figure 7. Showing the Geometric Relationship between the Feldspars and Mafic minerals with respect to Solution and Factional Crystallization

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Appendix

Chemical Expression of All Silicate Minerals Used in Madeling

A1: Olivine Family: ($X_{2-x}Y_x$)Z

(a) Magnesium rich olivine
- Forsterite: Mg_2SiO_4
- Chrysolite: $(Mg_3Fe)Si_2O_8$
- Hyalosiderite: $(Mg_6Fe_4)Si_{10}O_{20}$

(b) Intermediate
- Monticellite: $(MgFe)SiO_4$

(c) Iron rich olivine
- Fayalite: Fe_2SiO_4
- Ferrohortonolite: $(MgFe_3)Si_2O_8$
- Hortonolite: $(Mg_4Fe_6)Si_{10}O_{20}$

(d) Spinel group
- Chrom spinel: Cr_2AlO_4
- Ruby: Mg_2AlO_4
- Magnetite: Fe_3O_4

A2: Pyroxene Family: ($X_{1-x}Y_x$)Z

(a) Magnesium rich pyroxene
- Enstatite: $(Mg_2Si_2O_6)$
- Bronzite: $(Mg_7Fe_3)Si_{20}O_{60}$
- Hypersthene: $(Mg_{13}Fe)Si_{20}O_{60}$

(b) Intermedite pyroxene
- Hypersthene: $(MgFe)Si_2O_6$

(c) Iron rich pyroxene
- Ferrosilite: $FeSi_2O_6$
- Eulite: $(MgFe_{13})$ $Si_{20}O_{60}$
- Ferrohypersthene: (Mg_7Fe_{13}) $Si_{20}O_{60}$

(d) Calcium rich pyroxene
- Diopside: $(CaMg)Si_2O_6$
- Hendenbergite: $(CaFe)Si_2O_6$

(e) Sodium rich pyroxene
- Aegirine: $(NaFe)Si_2O_6$

A3: Amphibole Family: ($X_{7-x}Y_x$)Z

(a) Magnesium rich amphibole
- Kupfferite:Mg7Si8$O_{22}(OH)_2$
- Cummingtonite: (Mg_9Fe_5) $Si_{16}O_{44}(OH)_4$

(b) Intermediate amphibole
- Hornblend: (Mg_3Fe_4) $Si_8O_{22}(OH)_4$

(c) Iron rich mineral
- Grunerite: $Fe_7Si_8O_{22}(OH)_2$
- Anthophyllite: (Mg_5Fe_9) $Si_{16}O_{44}(OH)_4$

(d) Calcium rich amphibole
- Tremolite: (Mg_5Ca_2) $Si_8O_{22}(OH)_2$
- Actinolite: : (Fe_5Ca_2) Si_8 $O_{22}(OH)_2$

(e) Sodium rich amphibole:
- Rebeickite: $Na_2Fe_2^{+3}Fe_3^{+2}$ $Si_8O_{22}(OH,F)_2$
- Pargasite: $(A_2B)(X_4Fe)(Q_6.N_2)O_{22}(OH)_2$

A = Ca, B =Na, X = Mg, Q = Si, N = Al

- Ferrohastingsite: $(A_2B)(X_4Fe)(Q_6.N_2)O_{22}(OH)_2$, Y = Fe
- Arfvedsonite: $(A_{3-x}B_x)(X_4Y)(M_{8-y}N_y)$ $O_{22}(OH)_2$
  :$(Na_5Ca)(Mg, Fe, Al)_{10}(Si_{15}Al)$ $O_{22}(OH)_2$

A4: Mica Family: ($X_{6-x}Y_x$)Z_2

(a) Magnesium rich mica
- Phlogopite:$K_2Mg_6Al_2Si_6O_{20}(OH)_4$

(b) Intermediate mica
- Biotite: $K_2(MgFe)_6Al_2Si_6O_{20}(OH)_4$

(c) Iron rich mica
- Lepidomelane: $K_2Fe_6Al_2Si_6O_{20}(OH)_4$

(d) Potassium rich mica
- Muscuvite: $K_2Al_4Si_8O_{20}(OH)_4$

(e) Sodium rich mica
- Paragonite: $Na_2Al_4Si_8O_{20}(OH)_4$

(f) Lithium rich mica
- Lepidolite
- Zinwaldite

A5: Felsic Mineral: $[(X_{1-x}Y_x)(A_{4-y}B_y)]O_8$

(a) Calcium rich plagioclase:
- Anorthite: $CaAl_2Si_2O_8$
- Bytownite: $NaCa_9Al_{19}Si_{21}O_{80}$
- Labradorite: $Na_3Ca_7Al_9Si_{11}O_{80}$

(a) Calcic-Sodic feldspar
- Feldspar: $(NaCa)Al_3Si_5O_{16}$

(b) Sodium rich plagioclase
- Albite: $NaAlSi_3O_8$
- Oligoclase: $NaCaAl_{11}Si_{29}O_{80}$
- Andesine: $Na_7Ca_3Al_{13}Si_{27}O_{80}$

(c) Feldsparthoid
- Nepheline: $NaAlSiO_4$
- Leucite: $KAlSi_2O_6$
- Jadeite: $NaAlSi_2O_6$

A6: Oxide Minerals

(a) Iron-Titanium Oxide: $[(X_{3-x}Y_x)O_4 : (X_{2-x}Y_x)O_3]$
- Magnetite: Fe_3O_4
- Haematite: Fe_2O_3
- Wustite: FeO

(b) Titanium rich Oxide
- Ilmenite: $FeTiO_3$
- Ulvospinel: Fe_2TiO_4
- Rutile: TiO_2
- Ilmeno-rutile: $FeTi_2O_4$
- Pseudobrookite: Fe_2TiO_5

(c) Coltan: $ZX_{5-x}Y_xO_6$
: $ZX_{5-x}Y_xO_6$
: $ZX_{4-x}Y_xO_6$
: $ZX_{3-x}Y_xO_6$
: $ZX_{2-x}Y_xO_6$

- Columbite: $FeNb_5O_6$
  : $FeNb_5O_6$
  : $FeNb_4O_6$
  : $FeNb_3O_6$
  : $FeNb_2O_6$
- Tantalite: $FeTa_5O_6$
  : $FeTa_4O_6$
  : $FeTa_3O_6$
  : $FeTa_2O_6$

A7: Hydrocarbons

(a) Alkane: $X_{n}\text{-}Y_{2n+2}$
- Methane: CH_4
- Ethane: C_2H_6
- Propane: C3H8
- Butane: C4H10