Modification of Bowen’s Reaction Series of Rock Forming Minerals Using Mathematical Mapping Method and Models with Respect to Thermodynamic Principles

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Abstract

During sequence of crystallization, the crystallization of rock forming minerals from magma $\mathfrak{R}$, was mapped using mathematical mapping method and Bowens and Goldschmidt combined models were developed under thermodynamic conditions. The problem involving the distribution of chemical elements and elemental substitutions in rock during crystallization of magma $\mathfrak{M}$ from the beginning of crystallization to the end of crystallization was mathematically resolved using Bowen’s and Goldschmidt combined model. “This research was focused on the modification of Bowen’s reaction series by election and elimination of minerals under thermodynamic change into Bowen’s and Goldschmidt combined model using mathematical mapping method during crystallization of magma”**.** Finally, findings have shown that, by using mathematical mapping method, two reactions series were propounded to define the sequence of crystallization of rock forming minerals during crystallization of magma. These two reactions series were Polymerization and Isomorphous reactions series. In mathematical context, it was concluded that continuous and discontinuous reactions series in Bowen’s Model were replaced by both Polymerization and Isomorphous reactions series when both Bowen and Goldschmidt concepts were used for concise explanation of rock forming minerals from the beginning to the end of crystallization and with these combined concepts, Bowen’s reaction series was modified. This modification would help the ‘beginners’ especially students of Earth sciences such as Geology, Mineralogy, Petrology and other chemical sciences such as Geochemistry and Petroleum Geology to have clear understanding of rock forming minerals from the Magma $\mathfrak{R}$.

Keywords

Bowen's Reaction Series, Rock forming Materials, Mathematical Mapping Method, Thermodynamic Principles

1. Introduction

The aim of this research focuses on the modification of Bowen’s reaction series of rock forming minerals by application of mathematical mapping method using combined Bowen and Goldschmidt concepts under thermodynamic conditions

Geometrically, several minerals set in matrices and aggregate themselves in coordinate to form rocks and chemically elements of several properties bonded together to form minerals, which by definition is inorganically in composition with distinct chemical composition. Goldschmidt classified these elements according to their class of materials such as siderophile, lithophile and chalcophile elements as well as atmophile elements. But these elements are arranged in periodic table according to size and atomic mass. According to modern periodic law, atomic number increases from left to right and from top to bottom of periodic table. Therefore the size and electronegativity of elements increase across the period and the size increases with decreasing electronegativity down the group. This pattern of modern periodic law would be used to explain how ions would enter into the lattice of a growing crystal during crystallization of silicate magma as a function of size and electronegativity. During crystallization of magma, temperature falls, with increasing content of silicon in the magma. This forms two series of reactions (Bowen’s), one by interaction with magma, with the first mineral formed, to form minerals of different chemical composition, but with the same crystallographic structure. This is the case of solid solution in the magma. While the other, the first crystal to form, initially interact with the magma to form a solid with different crystallographic structure, and distinct chemical composition. This is the case of fractional crystallization in the magma.

The term solid solution and fractional crystallization are used simultaneously in this research to define the two series of reactions as stipulated by Bowen’s reaction series.

The problem involving the distribution of chemical elements and elemental substitutions in rock during crystallization of magma $\mathfrak{M}$, from the beginning of crystallization to the end of crystallization would be resolved to some an extent using Goldschmidt and Bowen’s concept with respect to electronegativity and radius ratio, which states that, for ions of various electronegativities and sizes to compete for the space in a lattice of a growing crystal;

The one with more electronegative would preferentially enter into the lattice of the growing crystal of silicate radical because the nucleus of silicate radical is electronegative provided modern periodic law is obeyed and

2. Literature Review

2.1 Bowen's Reaction Series

Bowen, uses two series of reactions to explain the evolution of igneous rocks from basaltic melt. Bowen's reaction principle a concept, first propounded in 1928 by Norman Bowen, which explains how mineral can respond to changing equilibrium conditions when a magma is cooled, by either a continuous diffusing - controlled exchange of elements with the magma or discontinuous melting of the material [1].

2.2 Rules Guiding the Distribution and Elemental Substitution of Chemical Elements in Rock

The periodic law was developed independently by Dmitri Mendeleev and Lothar Meyer in 1869 to arrange the elements by their mass and proposed that certain properties periodically reoccur [2].

Goldschmidt proposed his Classical general rules to explain the distribution of the elements, in which ions of similar size and charge substitute themselves [3].

The complementary use of the concept of electronegativity in order to understand the distributions of the chemical elements that could not be explained completely with the Goldschmidtian rules, especially when the minerals being investigated had high percentages of covalent bonding was proposed [4].

Goldschmidt rules 2 and 3, were modified which states that the site has a preferred radius of Ion (r) which eaters mostly easily, for ions of the same charge, those which are closest in radius to enter most easily, ions which are larger or smaller are discriminated against [4]

Nickold proposed that the tree principal factors (ionic size, Ionics charge and electronegativity) be expressed in a single function that would not result in the dichotomous predictions [5].

3. Methology

3.1 Thematic Map

Thematic analysis is a method for identifying, analyzing and reporting patterns (themes) within data [6].

Thematic Thermal Magmatic Contour Map

A Thematic Thermal M agmatic Contour Map is designed purposely for this work to represent the intensity of "thermal" occurrence within a data set or system (magma). This map is connected to the "Particles of the Universe" and "the Electromagnetic Spectrum" which absorbs or emits energy as a "Photon" as well as Silica polymorphs such as Cristoballite, tridymite and Quartz as shown in Figure (1) below. This connection is due to thermal occurrence in the map which represents Magmas, because every object in its highest potential energy at high temperature absorbs certain amounts of energy from the universe, but under its low potential energy at its lowest energy emits the same quantum amount of energy to the universe inform of "LIGHT" called "PHOTON", This type of map uses colour to represent the thermal intensity of each point in the complex plane of magma $\mathfrak{R}$, and is a function of atomic lattice in the magma. The atomic lattice in the complex plane of magma $\mathfrak{R}$ consists of the constituent atoms or elements that make up that lattice. In this case, each atom in the lattice has a minimum of free energy (E), which depends on the temperature within their stability limit as shown in Figure (1) below.

Generally, the aim of this map is to visualize and analyze point data by transforming the points into a thermal grid. Each resulting grid cell is assigned a numerical value in degrees Celsius that is determined by the proximity of nearby points, optionally, measuring each point using geological thermometer as shown in figure (1) below. This map uses colours that represent the values from high to low especially from high temperature minerals to low temperature minerals.

If an atom is heated or subjected to an electrical discharge, it absorbs energy which is subsequently emitted as radiation. Therefore atom in its excited state absorbs certain equivalent amount of energy from the universe as showed in figure (1) in which $E = fh$ emits the same amount of energy when it returns back to the ground state level to the universe, either within the visible or invisible portion of the electromagnetic spectrum in form of light called photon. For example from the Figures (2) and (3), sodium ion in its excited state absorbs certain frequency of colour from the universe and when returns to its ground state level tends to emits the same amount of frequency of light which give rise to characteristic yellow flame coloration of sodium atom in the invisible potion of electromagnetic spectrum and the colour is observed when heated in a flame of bursen burner. In atomic spectroscopy, sodium atom shows two lines of emission spectra which correspond to the wavelengths of 589nm and 589.6nm. If a hydrogen is subjected to an electric discharge, some of the hydrogen atoms are produce, which emits light in the visible region of the electromagnetic spectrum. This can be studied using spectroscopy and can be found to comprise series of lines of different wavelengths. Four lines can be seen by the eye, many more are observed photographically in the ultraviolet region.

The colour that one sees when looking at a mineral or gemstones are due to respond of that person's eye to the energies of the light, the emission spectrum of the illumination. Light is a form of energy (electromagnetic energy) and white light is a mixture of all of the visible energies or wavelengths. In order for a mineral to cause colour from the white light it has to somehow perturb the balance of the light energy. Kurt Nassau has separated the causes of colour into 15 mechanisms based on 5 physical groupings. While there are some colour mechanisms that depend on direct emission of certain colour, most of the mechanisms we are interested in depend on the ability of minerals to preferentially absorb certain energies of light. When these energies are removed from the white light the mineral is colored by the complementary colour.

Light absorption by the electrons of transition metal (or rare earth element) atoms, either as major portion of the mineral chemistry or impurities is one of the most important and well known of the coloring mechanisms. Most rock forming elements have electronic structures which mitigate against causing colours. On the other hand transition metals and REE's have electrons which can be excited to open, higher energy levels. The electrons gain the necessary energy for the excitation by absorbing a particular energy (colour) from white light and thus cause the mineral to show the complementary colour. The prime examples of this mechanism are Rubies, Emiralds and Alexandrites.

The second most important and common colouring mechanisms is intervalence charge transfer. This occurs when a valence electron from one atom transfer to the structure of close-by atom (often completely different elements) again by absorbing just the amount of energy needed to make the transfer. Examples include sapphire, lapis and amazonite.

This is the thematic thermal Contour Map of different magmas of the same source, from the basic magma to acidic magma. Figure (1), depicts the temperatures of different types of rocks from different magmas. These magmas have the same principal origin called Basaltic magma and when subjected to cooling under isobaric condition, the following magmas can be mapped as;

- Basic magma
- Acidic Magma
- Mafic magma
- Felsic magma

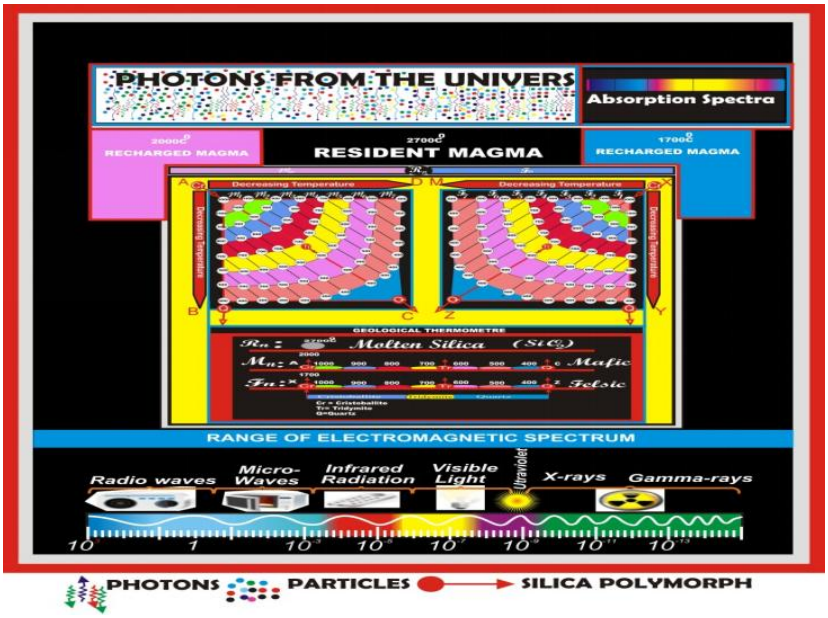

Figure 1. Thematic Thermal Contour Map with Geological thermometer

3.2 Mathematical Mapping

A mathematical relation such each element of a given set (the domain of the function) is associated with an element of another set (the range of the function and it is a rule which assigns an element **x** in a set **A** to another unique element **y** in set **B**.

This is the application of mathematics to mapping of magmas, minerals and rocks with respect to temperature under isobaric condition. Each temperature in the magmas corresponds to each mineral. The process in which each temperature corresponds to each mineral and rock in the magma under thermodynamic change is called Mathematical Mapping in Geology.

Given that is the mafic magma and $\mathbf{F_m}$ is the felsic magma, then;

$\mathcal{M}_m$ = b, c, d, f, g, h, k

Ƒ_m = a, e, i, o, u, w, y

b, c, d, f, g, h, k are set of element in mafic magma, such that, b, c, d, f, g, h, $k \in \mathcal{M}_m$

a, e, i, o, u, w, y, are set of element in mafic magma, such that, a, e, i, o, u, w, $y \in \text{Ƒ}_m$

{b, c, d, f, g, h, k} + {a, e, I, o, w, y} are set of elements in the recharged magma, $\mathfrak{W}$

Therefore, {b, c, d, f, g, h, k} + {a, e, I, o, w, y}

Therefore, $\mathcal{M}_m \longrightarrow \text{Ƒ}_m$, that is, $\mathcal{M}_m$ Maps Ƒ_m as given below;

$b \longrightarrow a$: 2000^0C

$c \longrightarrow e$: 1000^0C

$d \longrightarrow i$: 900^0C

$f \longrightarrow o$: 800^0C

$g \longrightarrow u$: 7000C

$h \longrightarrow w$: 600^0C

$k \longrightarrow y$: 500^0C

$z \longrightarrow 400^0C$

3.3 Resident Magma $\mathfrak{R}$ with Homogeneous Nucleation of Atoms at High Temperature and Absorption Spectra

In a new model of crystallization in magma, Toramaru, A., and Kichise, T used a new numerical experiments to study the influence of different cooling rates and classical nucleation theory parameters on the crystal number density measured under constrained conditions in the laboratory experiments and identifies that, the nucleation rate as the main parameter that controls the formation of crystal. In this condition it is interesting that the crystal growth rate is inversely proportional to the time, even if the diffusion limited growth is inversely proportional to the square root of time [7].

Andrew, B.J., and Befuse, K, S developed a numerical model which is supersaturation nucleation and growth of plagioclase that predicts the nucleation and growth of plagioclase crystals in a decompressing magma as a function of time and that growth rates are used to grow the existing crystals, where nucleation adds new crystals [8].

In any chemical system, such as silicate melt $\mathfrak{R}$, the atoms are constantly moving about, bumping into neighbors at high temperature and absorb certain frequencies of energy in the electromagnetic spectrum as shown Figure 4 below.

The embryos cannot grow if the mineral has a higher Gibbs free energy of formation (less stable) than the same elements remaining in solution, as part of the melt or tied up in other minerals. The embryos will therefore break apart and the constituent atoms/ions will return to the lower energy configuration represented by the melt, solution or other minerals.

At some point, however, the temperature, pressure, or fluid composition (or all three) will change so that the crystalline structure of a new mineral represents a lower energy configuration for the elements. This does not however, mean that all of the embryos will immediately begin to grow into mineral grains. This complication factor is the surface energy of the embryos. This destructive chemical bond at the surface of each embryo represents a higher energy configuration. The magnitude of surface energy is proportional to the surface area [9].

For homogeneity of magma $\mathfrak{R}$ as shown in Figure (2) below, all elements are in their highest potential energy at their highest temperatures which is linked to the particles of Universe and they absorb certain frequencies of energies from the Universe and each of these energies is discrete in form of chunks or packets, called energy quantization. When atoms or ions in the magma are in their excited state at their highest temperature, they absorb energies, some of which can be observed in the visible portion of electromagnetic spectrum and others in the invisible portion of the electromagnetic spectrum depending on the energy of the vibration of the atoms. Each of these atoms absorb discrete packet of energy in form of light called Photons. If an atom is heated or subjected to an electrical discharge, it absorbs energy which is subsequently emitted as radiation. Therefore atom in its excited state absorbs certain equivalent amount of energy from the universe as showed in figure 1 in which $E = hf$ and emits the same amount of energy when it returns back to the ground state level to the universe, either within the visible or invisible portion of the electromagnetic spectrum in form of light called photon [10].

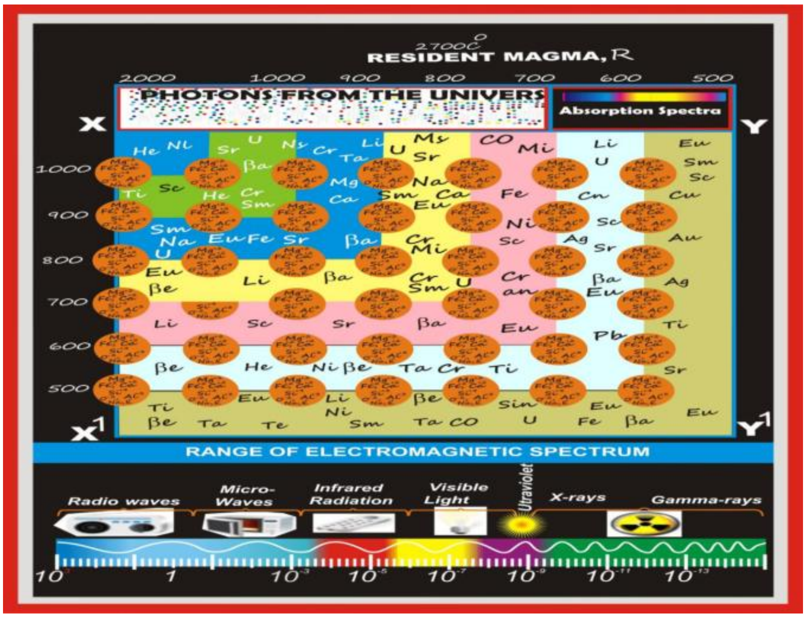

Figure 2. Resident Magma $\mathfrak{R}$ with Homogeneous Nucleation of Atoms at High Temperature and Absorption spectra

Where;

$(XX^1)(YY^1)$ = Homogeneous Magma $\mathfrak{R}$ at 2700^0C.

3.4 Recharged Magma $\mathfrak{W}$ and Heterogeneous Nucleation

When temperature of the melt (Molten Silica) $\mathfrak{R}$ drops, a variety of chemical combinations of atoms will nucleate and is called heterogeneous nucleation as shown in Figure (3) below.

The combinations of the nucleated atoms from the melt $\mathfrak{W}$ are called embryos, and will by chance have the composition and structure of a mineral that could crystallize from the melt as shown in Figure 5 below. Most of the embryos will be small, consisting of only a few atoms, but some will be large. Whether these embryos will ultimately grow to form mineral grains when the equilibrium is above the critical point and the size of the embryo is above the critical size of the embryos are dependent on both the stability and the critical size of the growing crystals from the embryos consist of several atoms.

If an existing mineral XX^1Y which is the Basic, has a surface or structure that is similar to a new mineral YY^1X^1 which is Acidic, the existing mineral, can serve as the nucleus for growth as show in Figure 3. The need to form embryos is largely eliminated. Once nuclei are stable, growth must progress by adding atoms/ions to the surface of the crystal. This faces kinetic problems similar to nucleation.

When magma is recharged by dropping the temperature of molten silica $\mathfrak{R}$, the magma becomes recharged $\mathfrak{W}$ and partially electrolytic, which is separated into Basic (XX^1Y) and Acidic (YY^1X^1) magmas, therefore with the heterogeneous nucleation, new minerals would nucleate by taking advantage of the structure of an existing mineral, which is Basic to another mineral which is Acidic as shown Figures (3) and these express the elimination of minerals from the melt and election of minerals into the melt under thermodynamic change as shown in Figures (4, 5, 6) below. Some of the energetic problems associated with homogeneous nucleation which would nucleate Basic mineral (XX^1Y) are avoided when forming Acidic mineral (YY^1X^1) and therefore emits no new energy inform of light to the universe in the visible portion of electromagnetic spectrum as shown in figure 5 below.

When Atoms lose energies as temperatures of molten silica dropped, they return to their ground state level and then emit the same amount of energies absorbed to the universe inform of lights called Photons either in the visible or invisible portion of electromagnetic spectrum as shown in in Figure (3) below.

For example from the figure 4, sodium ion (Na^{+1}) in its excited state absorbs certain frequency of colour from the universe as shown in Figure (3) and when returns to its ground state level tends to emits the same amount of frequency of light which give rise to characteristic yellow flame coloration of sodium atom in the invisible potion of electromagnetic spectrum and the colour is observed in the visible portion of electromagnetic spectrum when heated in a flame of bursen burner as shown in Figure (3) below. In atomic spectroscopy, sodium atom shows two lines of emission spectra which correspond to the wavelengths of 589nm and 589.6nm.

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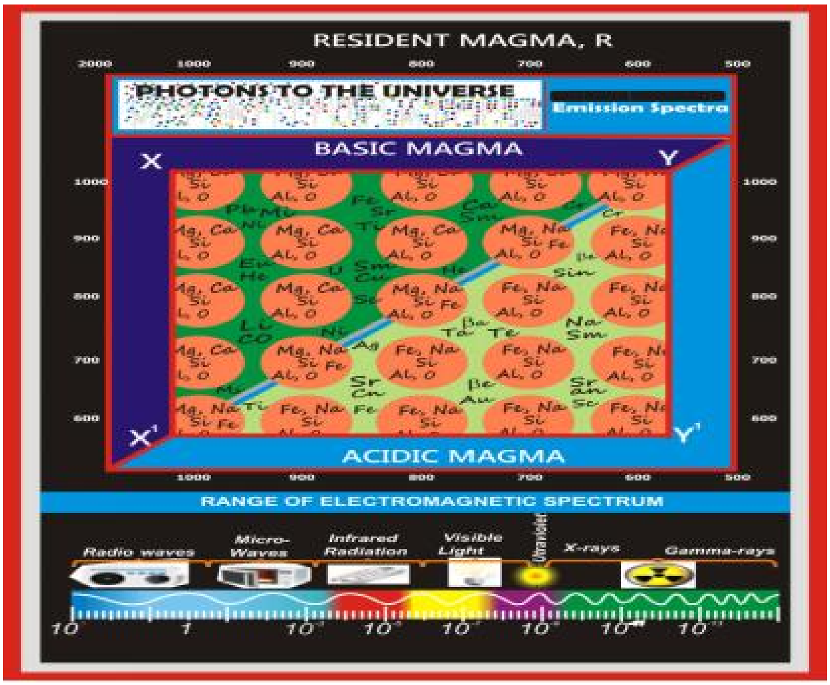


Figure 3. Recharged Magma $\mathfrak{W}$ with Heterogeneous Nucleation and Emission spectra

Where;

XX^1Y = Basic magma

YY^1X^1 = Acidic magma

4. Results and Analyses

The results obtained in the research were basically from literatures of different Journals and Textbooks.

The temperatures of minerals in rocks were obtained and certain parameters such as geological thermometres and mathematical mappings under thermodynamic change were used to analyse them and placed them in a geometric planes as sown in Figures, 4, 5, 6, 7, and 8.

According to Bowen's experiments, magmatic temperatures range from $1300\ ^{\circ}C - 800^{\circ}C$ for mafic and $1500^{\circ}C - 1100^{\circ}C$ *for felsic*

4.1 Analyses of Magma under Thermodynamic Change

Figure (1) above, is a Thematic Thermal Contour Map used to depict temperatures of different points in complex plane of Magma $\mathfrak{R}$, and each temperature corresponds to lattice space which consists of ions or atoms with unique quantum amount of energy inform of particles. The lattices in the plane under thermodynamic change form minerals of equivalent amount of energy. These minerals are represented in the plane with corresponding temperatures in the map. In mathematical context, when minerals are elected in a plane that correspond with the temperatures of formation, it is called mathematical mapping in Geology.

Geometrically Z, is a plane in the Magma $\mathfrak{W}$ that represents two points X and Y axes, where X is horinzontal axis called "Polymerization" series and *Y*, is vertical axis called "Isomorphous" series, as shown in Figure (4) below.

In order to map the magmas, minerals and rocks from crystallizing magma $\mathfrak{R}$, as seen in the Map as represented in Figure 1 below, the following geological tools are used;

- Molten Silica : 2700°*C*
- Silica polymorphs: 2700-400°*C*: Horizontal
- Mafic: 2000-400°*C*: Vertical axis
- Felsic: 1700-400°*C*: Vertical axis

4.2 Mapping of Magma with Respect to Gibbs Energy and Le-Chaterlier Principle

Given that $\mathfrak{R}$ is complex plane of resident magma and $\mathfrak{W}$ is the recharged magma, geometrically, under thermodynamic change vowel and consonant alphabets will be used to represent the coordinate of both feldspar and mafic minerals that would crystallize from the recharged melt $\mathfrak{W}$ during crystallization as shown in the Figure 2. The process of representing this in a plane of magma is called Mathematical transformation of magma from the beginning to the end of crystallization. From figure 6 below, the set of mafic minerals and felsic minerals are mathematically given below.

Given that $\mathcal{M}_{\mathrm{m}}$ is the mafic mineral and *fm* is the felsic mineral as shown in thematic Thermal magmatic contour map in figure 1 and

$\mathcal{M}_{\mathrm{m}} = b, c, d, f, g, h, k$

$\mathcal{F}_m = a, e, i, o, u, y, w$

∴ Setting, the addition of $\mathcal{M}_m$ & $\mathcal{F}_m$ *equal to W* as shown in equation (1);

As $\mathfrak{R} \longrightarrow \mathfrak{W}$.

This means that both $\mathcal{M}_{\mathrm{m}}$ and Ƒm remain at equilibrium with the melt $\mathfrak{W}$, because the degree of freedom for the three phases to coexist together without elimination of any of the phases is zero, that is, no condition is specified according phase relationship by Gibbs.

For rock forming minerals to form from the melt $\mathfrak{W}$ which set in matrix to form rock of certain composition, both the mafic and felsic minerals must mathematically be set at equilibrium as shown in equations (1) and (2) below and the process will continue throughout the time of crystallization.

Since mafic $\mathcal{M}_{\mathrm{m}}$ elements are contained in the magma $\mathfrak{W}$ and felsic elements Ƒm are contained in the magma $\mathfrak{W}$

Therefore mathematically, $\mathcal{M}_{\mathrm{m}} \in \mathfrak{W}$ and $f_{\mathrm{m}} \in \mathfrak{W}$ are represented as;

$\mathcal{M}_{\mathrm{m}} + \mathcal{F}_m = W$ (1)

$\mathcal{M}_{\mathrm{m}}$ + Ƒm = (b, c, d, f, g, h, k) + (**a, e, i, o, u, y, w**) (2)

(**b, c, d, f, g, h, k**) + (**a, e, i, o, u, y, w**) = $\mathfrak{W}$ (3)

(**b, c, d, f, g, h, k**) + (**a, e, i, o, u, y, w**) $\in \mathfrak{W}$

(**b, c, d, f, g, h, k**) ∈ $\mathcal{M}_{\mathrm{m}}$ + (**a, e, i, o, u, y, w**) ∈ Ƒm = (b, c, d, f, g, h, k) + (**a, e, i, o, u, y, w**) $\in \mathfrak{W}$

Therefore; (**b, c, d, f, g, h, k**) and (**a, e, i, o, u, y, w**) are sets of elements that are in the melt, $\mathfrak{W}$

Where, (**b, c, d, f, g, h, k**) are sets of element in mafic magma $\mathcal{M}_{\mathrm{m}}$ and (**a, e, i, o, u, y, w**) are sets of element in felsic magma Ƒm.

Analogy 1: **First Order of Crystallization in the Melt $\mathfrak{W}$ under Thermodynamic Condition**

Given that **b**$_{n,}$ is an element in mafic magma $\mathcal{M}_{\mathrm{m}}$ and **a,** is an element in felsic magma **Ƒm at 2000^{0}C,** then;

$b_{n,} \in \mathcal{M}_{\mathrm{m}}$ **Λ** $a \in$ Ƒm

∴ $\mathcal{M}_{\mathrm{m}} + \mathcal{F}_m = \mathrm{b_n} + \mathrm{a}$

$\mathfrak{W} = \mathbf{b_n} + \mathbf{a}$ (4)

$\mathfrak{W} = \mathbf{b_n} + \mathbf{a}$, is the First Order of Crystallization in the melt $\mathfrak{W}$ under thermodynamic change such that;

For $n = 1$

$\mathfrak{W} \mathbf{= b_1 + a}$**,** as shown in in Figure (6), Fraction 1, Order 1

$\mathfrak{W} = b_1 + a$ (5)

This means that both $\underline{\boldsymbol{b}}_1$ and $\underline{\boldsymbol{a}}$ are in a state of equilibrium with the melt $\mathfrak{W}$ and are elected into the first order of crystallization as shown in Table (4) and Figure (6) below.

So that;

$\mathbf{b_1}, \in \mathcal{M}_m : \mathbf{b_1}$ = basic olivine (Mg>Fe)

$\mathbf{a}, \in \mathcal{F}_m$: $\mathbf{a}$ = basic plagioclase(Ca>Na)

$\underline{\mathbf{a}}$ **maps** $\underline{\mathbf{b}}_1$ in the melt $\mathfrak{W}$ at 2000^0C during crystallization of magma under thermodynamic change as gives below;

$\underline{\mathbf{a}} \longrightarrow \underline{\mathbf{b}}_1$, at 2000^0C.

Therefore;

One plagioclase feldspar maps one basic olivine in the melt and this mapping is called one to one, mapping in geology.

Therefore from the analogy above, $\underline{\boldsymbol{b}}_1$ and $\underline{\boldsymbol{a}}$ are the first two crystal phases to be crystallized from the melt in which case $\mathcal{M}_m$ *and* f_m are of different phases of mafic and felsic magmas from which these minerals are differentiated.

Analogy 2: Second Order of Crystallization in the Melt $\mathfrak{W}$ under Thermodynamic Change

In order to promote to the next second order of crystallization, the first two earlier phases $\underline{\mathbf{a}}$ and $\underline{\mathbf{b}}_1$ of first order of crystallization must completely be eliminated from the melt $\mathfrak{W}$, so that the two new formed phases $\underline{\mathbf{e}}$ and $\underline{\mathbf{c}}_1$ with the melt $\mathfrak{W}$, would be at equilibrium at the new temperature of $\mathbf{1000^0c}$ which would neutralize the cooling effect of the new formed system composing the phases $\underline{\mathbf{e}}$ and $\underline{\mathbf{c}}_1$ with the melt $\mathfrak{W}$ by releasing more heat to the surrounding, according to Le-chaterllier principle, therefore from equation (6) beolw;

At, $\mathfrak{W} = 0$, then;

$b_n + a = 0$ (6)

$n = 2$

$b_2 + a = 0$ (7)

Add equations (5) and (7) to eliminate $\underline{\mathbf{b}}_1$ and $\underline{\mathbf{a}}$**,** from the melt $\mathfrak{W}$ and elect $\underline{\mathbf{e}}$ and $\underline{\mathbf{c}}_n$ into the melt $\mathfrak{W}$ of second order of crystallization at $\mathbf{1000^0C}$ as given in the simultaneous equation below;

$b_1 + a = 0$

$b_1 + e = c_n$

Therefore eliminate b_1;

$$\begin{array}{l} b1+a=0 \\ \underline{\quad -b1+\ e=cn} \\ \quad a-e=-cn \end{array}$$

$\therefore a-e=-c$ (8)

$\underline{\mathbf{a}}$ and $\underline{\mathbf{b}}_1$ can be completely eliminated from the melt $\mathfrak{W}$, so that $\underline{\mathbf{e}}$ and $\underline{\mathbf{c}}_n$ can be elected into the second order of crystallization which is neutralized by the cooling effect at $\mathbf{1000^0C}$ and remain at equilibrium with the melt $\mathfrak{W}$ at $\mathbf{1000^0C}$

$a = -c_n + e$ (9)

From equation 4, of the first order;

$\mathbf{b_n} + \mathbf{a} = 0$

Put the value of a in equation (9) above, so that;

$b_2 - c_n + e = 0$

$\mathfrak{W} = \mathbf{b_2 - c_n + e}$ (10)

$\mathfrak{W} = \mathbf{b_2 - c_n + e}$ is the Second Order of Crystallization in the melt $\mathfrak{W}$ under thermodynamic condition.

For n = 1, then;

$b_2 - c_1 + e = 0$

$b_2 - c_1 + e = 0$ (11)

Since b_2 and c_1 are in equilibrium with the melt $\mathfrak{W}$ and the negative sign indicates that both the b_2 and c_1 are of different fractions while e is distributed across both the b_2 and c_1.

$\mathbf{b_2}, \mathbf{c_1} \in \mathcal{M}_m$ and $\mathbf{e} \in \mathcal{F}_m$

$\mathcal{M}_m + \mathcal{F}_m = 0$

$\mathcal{M}_m = b_2 - c_1$

$\mathcal{F}_m = e$

$c_1 + e = 0$ as in Figure (6), fraction (2), order (2)

$\mathfrak{W} = c_1 + e$ (12)

$b_2 + e = 0$ as in Figure (6), fraction (1), order (2)

$\mathfrak{W} = b_2 + e$ (13)

Mathematically, $\underline{b}_2$, and $\underline{c}_1$ are set of elements in mafic magma $\mathcal{M}_m$ such that, $\underline{b}_2$ is the basic olivine, $\underline{c}_1$ is basic pyroxene, and $\underline{e}$ is an element in felsic magma $\mathcal{F}_m$ such that $\underline{e}$ is the basic plagioclase at 1000^0C.

Therefore;

$\mathbf{b_2}, \mathbf{c_1} \in \mathcal{M}_m$: $\mathbf{b_2}$ = basic olivine (Mg>Fe)

:$\mathbf{c_1}$ = basic pyroxene (Mg>Fe)

$e \in \mathcal{F}_m$: $\mathbf{e}$ = basic plagioclase(Ca>Na)

So that, **$\underline{e}$ maps** $\underline{b}_2$, $\underline{c}_1$ in the melt $\mathfrak{W}$ at 2000^0C during crystallization of magma under thermodynamic change as gives below;

$\underline{e} \longrightarrow \underline{b}_2$, at 1000^0C.

$\underline{e} \longrightarrow \underline{c}_1$, at 1000^0C.

$\mathbf{b_2}, \mathbf{c_1}, \mathbf{e} \in \mathfrak{W} = 1000^0$C

Analogically, one plagioclase feldspar $\underline{e}$, maps one basic olivine $\underline{b}_2$, and one basic pyroxene $\underline{c}_1$, in the melt and this mapping is called one to two, mapping in Mathematics.

Therefore from the analogy above, $\underline{b}_2$, $\underline{c}_1$ and $\underline{e}$ are the second three crystal phases to be crystallized from the melt $\mathfrak{W}$ and elected into the Second Order of Crystallization in which case $\mathcal{M}_m$ *and* f_m are of different phases of mafic and felsic magmas from which these minerals are differentiated.

Analogy 3: Third Order of Crystallization in the Melt $\mathfrak{W}$ under Thermodynamic Change

As crystallization continues, and progresses to the next third order of crystallization under thermodynamic change, the second two earlier phases $\underline{e}$ and $\underline{c}_1$ of second order of crystallization must completely be eliminated from the melt $\mathfrak{W}$, so that the two new formed phases $\underline{i}$ and $\underline{d}_1$ with the melt $\mathfrak{W}$, would be at equilibrium at the new temperature of **1000^0C** which would neutralize the cooling effect of the new formed system composing the crystal phases $\underline{i}$ and $\underline{d}_1$ with the melt $\mathfrak{W}$ by releasing more heat to the surrounding, according to Le-chaterllier principle

Given that, the earlier phases of the second order of crystallization is shown in equation (14) below;

$c1+e=0$ (14)

As temperature drops, then the next order of crystallization is shown in equation (15) below;

$c1+i=dn$ (15)

Add equations (14) and (15) to eliminate $\underline{c}_1$ and $\underline{e}$**,** from the melt $\mathfrak{W}$ and elect $\underline{i}$ and $\underline{d}_n$ into the melt $\mathfrak{W}$ of third order of crystallization at **900^0C** as given in the simultaneous equation below;

$c1+e=0$

$c1+i=dn$

$c1 +e=0$

$$\begin{array}{l} (-)\ c1+i=dn \\ \hline e-i=-dn \end{array}$$

$\underline{\mathbf{e}}$ and $\underline{\mathbf{c}}_1$ can be completely eliminated from the melt $\mathfrak{M}$, so that $\underline{\mathbf{i}}$ and $\underline{\mathbf{d}}_n$ can be elected into the melt $\mathfrak{M}$ of third order of crystallization which is neutralized by the cooling effect at **900⁰C** and remain at equilibrium with the melt $\mathfrak{M}$ at **900⁰C**

$e = -d_n + i$ (16)

From equation (10) of the second order;

$\mathbf{b_n - c_n + e = 0}$ (17)

n = 3 for basic olivine

n = 2 for basic pyroxene

$b_3 - c_2 + e = 0$ (18)

Put the value of **e** in equation (18) above, so that;

$b_3 - c_2 - d_n + i = 0$

$\mathfrak{M} = \mathbf{b_3 - c_2 - d_n + i}$ (19)

$\mathfrak{M} = \mathbf{b_3 - c_2} - \mathbf{d_n} + \mathbf{i}$ is the Third Order of Crystallization in the melt $\mathfrak{M}$ under thermodynamic condition.

For n = 1 for basic plagioclase, then;

$b_3 - c_2 - d_1 + i = 0$

$b_3 - c_2 - d_1 + i = 0$ (20)

Since $\underline{\mathbf{b}}_\mathbf{3}$**,** $\underline{\mathbf{c}}_2$, $\underline{\mathbf{d}}_1$, and $\underline{\mathbf{i}}$ are in equilibrium with the melt $\mathfrak{M}$ and the negative sign indicates that both the $\underline{\mathbf{b}}_3$, $\underline{\mathbf{c}}_2$ and $\underline{\mathbf{d}}_1$ are of different fractions while $\underline{\mathbf{i}}$ is distributed across both the $\underline{\mathbf{b}}_3$, $\underline{\mathbf{c}}_2$ and $\underline{\mathbf{d}}_1$.

$\underline{\mathbf{b}}_3, \underline{\mathbf{c}}_2, \underline{\mathbf{d}}_1, \in \mathcal{M}_m\ \mathbf{\Lambda}\ \underline{\mathbf{i}} \in Ƒ_m$

$\mathcal{M}_m + Ƒ_m = 0$

$\mathcal{M}_m = b_3 - c_2 - d_1$

$Ƒ_m = i$

$\mathfrak{M} = b_3 + i$ (21)

$b_3 + i = 0$ as in Figure (6), fraction (1), order (3)

$\mathfrak{M} = c_2 + e$ (22)

$c_2 + e = 0$ as in Figure (6), fraction (2), order (3)

$\mathfrak{M} = d_1 + i$ (23)

$d_1 + i = 0$ as in Figure (6), fraction (3), order (3)

Mathematically, $\underline{\mathbf{b}}_3$, $\underline{\mathbf{c}}_2$ and $\underline{\mathbf{d}}_1$ are set of elements in mafic magma $\mathcal{M}_m$ such that, $\underline{\mathbf{b}}_3$ is the basic olivine, $\underline{\mathbf{c}}_2$ is basic pyroxene, $\underline{\mathbf{d}}_1$ is the basic amphibole and $\underline{\mathbf{i}}$ is an element in felsic magma $Ƒ_m$ such that $\underline{\mathbf{i}}$ is the basic plagioclase at 900⁰C.

Therefore;

$b_3, c_2, d_1 \in \mathcal{M}_m$: b_3 = basic olivine (Mg>Fe)

:c_2 = basic pyroxene (Mg>Fe)

:d_1 = basic amphibole (Mg>Fe)

$i \in Ƒ_m$: i = basic plagioclase (Ca>Na)

So that**, $\underline{\mathbf{i}}$ maps** $\underline{\mathbf{b}}_3$, $\underline{\mathbf{c}}_2$ and $\underline{\mathbf{d}}_1$ in the melt $\mathfrak{M}$ at 900⁰C during crystallization of magma under thermodynamic change as gives below;

$\underline{i} \longrightarrow \underline{\mathbf{b}}_3$, at 900⁰C.

$\underline{i} \longrightarrow \underline{\mathbf{c}}_2$, at 900⁰C.

$\underline{i} \longrightarrow \underline{\mathbf{d}}_1$, at 900^0C.

$\mathbf{b}_3, \mathbf{c}_2, \mathbf{d}_1, \mathbf{i} \in \mathfrak{W} = 900^0\text{C}$

Analogically, one plagioclase feldspar $\underline{\mathbf{e}}$, maps one basic olivine $\underline{\mathbf{b}}_3$, one basic pyroxene $\underline{\mathbf{c}}_2$, one basic amphibole $\underline{d}_1$, in the melt and this mapping is called one to three, mapping in Mathematics.

Therefore from the analogy above, $\underline{\boldsymbol{b}}_3$, $\underline{\mathbf{c}}_2$, $\underline{\mathbf{d}}_1$ and $\underline{\boldsymbol{i}}$ are the third four crystal phases to be crystallized from the melt $\mathfrak{W}$ and elected into the Third Order of Crystallization in which case $\mathcal{M}_m$ *and* f_m are of different phases of mafic and felsic magmas from which these minerals are differentiated.

Analogy 4: Fourth Order of Crystallization in the Melt $\mathfrak{W}$ under Thermodynamic

As crystallization continues, and progresses to the next fourth order of crystallization under thermodynamic change, the third two earlier phases $\underline{\mathbf{i}}$ and $\underline{\mathbf{d}}_1$ of third order of crystallization must completely be eliminated from the melt $\mathfrak{W}$, so that the two new formed phases $\underline{\mathbf{o}}$ and $\underline{\mathbf{f}}_1$ with the melt $\mathfrak{W}$, would be at equilibrium at the new temperature of **800^0C** which would neutralize the cooling effect of the new formed system composing the crystal phases $\underline{\mathbf{o}}$ and $\underline{\mathbf{f}}_1$ with the melt $\mathfrak{W}$ by releasing more heat to the surrounding, according to Le-chaterllier principle

Given that, the earlier phases of the second order of crystallization is shown in equation (24) below;

$d1+i=0 \qquad (24)$

As temperature drops, then the next order of crystallization is shown in equation (25) below;

$d1+o=fn \qquad (25)$

Add equations (24) and (25) to eliminate $\underline{\mathbf{d}}_1$ and $\underline{\mathbf{i}}$**,** from the melt $\mathfrak{W}$ and elect $\underline{\mathbf{o}}$ and $\underline{\mathbf{f}}_n$ into the melt $\mathfrak{W}$ of fourth order of crystallization at **800^0C** as given in the simultaneous equation below;

Given that;

$$d1+i\ =0$$

$$\underline{(-)\ d1+o=fn}$$

$$i-oi=-fn$$

$\underline{\mathbf{i}}$ and $\underline{\mathbf{d}}_1$ can be completely eliminated from the melt $\mathfrak{W}$, so that $\underline{\mathbf{o}}$ and $\underline{\mathbf{f}}_n$ can be elected into the melt $\mathfrak{W}$ of fourth order of crystallization which is neutralized by the cooling effect at **800^0C** and remain at equilibrium with the melt $\mathfrak{W}$ at **800^0C**

$i = -f_n + o \qquad (26)$

From equation (19) of the third order;

$\mathfrak{W} = \mathbf{b}_n - \mathbf{c}_n - \mathbf{d}_n + \mathbf{i}$

n = 4 for intermediate olivine

n = 3 for intermediate pyroxene

$\mathfrak{W} = b_4 - c_3 - d_2 + i \qquad (27)$

Put the value of i in (48) above, so that;

$\mathfrak{W} = \mathbf{b}_4 - \mathbf{c}_3 - \mathbf{d}_2 - \mathbf{f}_n + \mathbf{o} \qquad (28)$

Therefore;

$\mathbf{b}_4 - \mathbf{c}_3 - \mathbf{d}_2 - \mathbf{f}_n + \mathbf{o}$ is the Fourth Order of Crystallization in the melt $\mathfrak{W}$ under thermodynamic condition.

For n = 1 for basic black mica, then;

$b_4 - c_3 - d_2 - f_1 + o = 0$

$b_4 - c_3 - d_2 - f_1 + o = 0 \qquad (29)$

Since $\underline{\mathbf{b}}_4$, $\underline{\mathbf{c}}_3$, $\underline{\mathbf{d}}_2$, $\underline{\mathbf{f}}_1$ and $\underline{\mathbf{o}}$ are in equilibrium with the melt $\mathfrak{W}$ and the negative sign indicates that both the $\underline{\mathbf{b}}_4$, $\underline{\mathbf{c}}_3$, $\underline{\mathbf{d}}_2$, and $\underline{\mathbf{f}}_1$ are of different fractions while $\underline{\mathbf{o}}$ is distributed across all fractions of $\underline{\mathbf{b}}_4$, $\underline{\mathbf{c}}_3$, $\underline{\mathbf{d}}_2$, and $\underline{\mathbf{f}}_1$

$\underline{\mathbf{b}}_4, \underline{\mathbf{c}}_3, \underline{\text{d}}_2, \text{f}_1 \in \mathcal{M}_m \ \Lambda\ \underline{\mathbf{o,}} \in \text{Ƒ}_m$

$\mathcal{M}_m + \text{Ƒ}_m = 0$

$\mathcal{M}_m = b_4 - c_3 - d_2 - f_1$

$\text{Ƒ}_m = o$

$\mathfrak{W} = b_4 + o$ (30)

b_4 + o = 0 as in Figure (6), fraction (1) and order (4)

$\mathfrak{W} = c_3 + o$ (31)

c_3 + o = 0 as in Figure (6), fraction (2) and order (4)

$\mathfrak{W} = d_2 + o$ (32)

d_2 + o = 0 as in Figure (6), fraction (3) and order (4)

$\mathfrak{W} = f_1 + o$ (33)

f_1 + o = 0 as in Figure (6), fraction (4) and order (4)

Mathematically, $\underline{\mathbf{b}}_4$, $\underline{\mathbf{c}}_3$, $\underline{\mathbf{d}}_2$, and $\underline{\mathbf{f}}_1$ are set of elements in mafic magma $\mathcal{M}_m$ such that, $\underline{\mathbf{b}}_4$ is the intermediate olivine, $\underline{\mathbf{c}}_3$ is intermediate pyroxene, $\underline{\mathbf{d}}_2$ is the intermediate amphibole f_1 is the basic black mica and $\underline{\mathbf{o}}$ is an element in felsic magma Ƒ_m such that $\underline{\mathbf{i}}$ is the basic plagioclase at 800^0C.

Therefore;

$\mathbf{b_4, c_3, d_2, f_1} \in \mathcal{M}_m$: b_4 = intermediate olivine (Mg=Fe)

:c_3 = intermediate pyroxene (Mg=Fe)

:d_2 = intermediate amphibole (Mg=Fe)

:f_1 = basic black mica (Mg>Fe)

$\mathbf{o} \in \text{Ƒ}_m$: o = intermediate plagioclase (Ca=Na)

So that, **$\underline{\mathbf{o}}$ maps** $\underline{\mathbf{b}}_4$, $\underline{\mathbf{c}}_3$ $\underline{\mathbf{d}}_2$ and $\underline{\mathbf{f}}_1$ in the melt $\mathfrak{W}$ at 800^0C during crystallization of magma under thermodynamic change as gives below;

$\underline{\mathbf{o}} \longrightarrow \underline{\mathbf{b}}_4$, at 800^0C.

$\underline{\mathbf{o}} \longrightarrow \underline{\mathbf{c}}_3$, at 800^0C.

$\underline{\mathbf{o}} \longrightarrow \underline{\mathbf{d}}_2$, at 800^0C.

$\underline{\mathbf{o}} \longrightarrow \underline{\mathbf{f}}_1$, at 800^0C.

$\mathbf{b_4, c_3, d_2, f_1, i} \in \mathfrak{W} = 800^0$C

Analogically, one plagioclase feldspar $\underline{\mathbf{o}}$, maps one intermediate olivine $\underline{\mathbf{b}}_4$, one intermediate pyroxene $\underline{\mathbf{c}}_3$, one intermediate amphibole $\underline{\text{d}}_2$, and one basic black mica $\underline{\mathbf{f}}_1$ in the melt and this mapping is called one to four, mapping in Mathematics.

Therefore from the analogy above, $\underline{\boldsymbol{b}}_4$, $\underline{\mathbf{c}}_3$, $\underline{\mathbf{d}}_2$, $\underline{\mathbf{f}}_1$ and $\underline{\boldsymbol{o}}$ are the fourth five crystal phases to be crystallized from the melt $\mathfrak{W}$ and elected into the Fourth Order of Crystallization in which case $\mathcal{M}_m$ *and* f_m are of different phases of mafic and felsic magmas from which these minerals are differentiated.

Analogy 5: Fifth Order of Crystallization in the Melt $\mathfrak{W}$ under Thermodynamic

As crystallization continues, and progresses to the next fifth order of crystallization under thermodynamic change, the fourth two earlier phases $\underline{\mathbf{o}}$ and $\underline{\mathbf{f}}_1$ of fourth order of crystallization must completely be eliminated from the melt $\mathfrak{W}$, so that the two new formed phases $\underline{\mathbf{u}}$ and $\underline{\mathbf{g}}_1$ with the melt $\mathfrak{W}$, would be at equilibrium at the new temperature of **700^0C** which would neutralize the cooling effect of the new formed system composing the crystal phases $\underline{\mathbf{u}}$ and $\underline{\mathbf{g}}_1$ with the melt $\mathfrak{W}$ by releasing more heat to the surrounding, according to Le-chaterllier principle

Given that, the earlier phases of the second order of crystallization is shown in equation (55) below;

f1+o=0 (34)

As temperature drops, then the next order of crystallization is shown in equation (56) below;

f1+u=gn (35)

Add equations (34) and (35) to eliminate $\underline{\mathbf{f}}_1$ and $\underline{\mathbf{o}}$**,** from the melt $\mathfrak{W}$ and elect $\underline{\mathbf{u}}$ and $\underline{\mathbf{g}}_n$ into the melt $\mathfrak{W}$ of fourth order of crystallization at **700^{0}C** as given in the simultaneous equation below;

Given that;

$f1 + o = 0$

$\underline{(-)\, f1 + u = gn}$

$o - u = -gn$

$\underline{\mathbf{o}}$ and $\underline{\mathbf{f}}_1$ can be completely eliminated from the melt $\mathfrak{W}$, so that $\underline{\mathbf{u}}$ and $\underline{\mathbf{g}}_n$ can be elected into the melt $\mathfrak{W}$ of fifth order of crystallization which is neutralized by the cooling effect at **700^{0}C** and remain at equilibrium with the melt $\mathfrak{W}$ at **700^{0}C**.

$o = -g_n + u$ (36)

From equation (28) above, of the fourth order;

$\mathfrak{W} = \mathbf{b_n - c_n - d_n - f_n + o}$ (37)

n = 5 for acidic olivine

n = 4 acidic pyroxene

n = 3 for acidic amphibole

n = for intermediate black mica

$\mathfrak{W} = b_5 - c_4 - d_3 - f_2 + o$ (38)

Put the value of o in (59) above, then;

$\mathfrak{W} = \mathbf{b_5 - c_4 - d_3 - f_2 - g_n + u}$ (39)

Therefore;

$\mathbf{b_5 - c_4 - d_3 - f_2 - g_n + u}$ is the **<u>Fifth Order of Crystallization</u>** in the melt $\mathfrak{W}$ under thermodynamic condition.

n = 1, for k-feldspar, then;

$\mathfrak{W} = b_5 - c_4 - d_3 - f_2 - g_1 + u$

$\mathfrak{W} = b_5 - c_4 - d_3 - f_2 - g_1 + u$ (40)

Since $\underline{\mathbf{b}}_5$, $\underline{\mathbf{c}}_4$, $\underline{\mathbf{d}}_3$, $\underline{\mathbf{f}}_2$, g_1 and $\underline{\mathbf{u}}$ are in equilibrium with the melt $\mathfrak{W}$ and the negative sign indicates that both the $\underline{\mathbf{b}}_5$, $\underline{\mathbf{c}}_4$, $\underline{\mathbf{d}}_3$, $\underline{\mathbf{f}}_2$ and g_1 are of different fractions while $\underline{\mathbf{u}}$ is distributed across all fractions of $\underline{\mathbf{b}}_5$, $\underline{\mathbf{c}}_4$, $\underline{\mathbf{d}}_3$, $\underline{\mathbf{f}}_2$ and g_1

$\underline{\mathbf{b}}_5, \underline{\mathbf{c}}_4, \underline{\mathbf{d}}_3, \mathbf{f}_2, \mathbf{g}_1 \in \mathcal{M}_m \land \underline{\mathbf{u}} \in Ƒ_m$

$\mathcal{M}_m + Ƒ_m = 0$

$\mathcal{M}_m = b_5 - c_4 - d_3 - f_2 - g_1$

$Ƒ_m = u$

$\mathfrak{W} = b_5 + u$ (41)

($b_5 + u = 0$ as in Figure (6), fraction (1), order 5)

$\mathfrak{W} = c_4 + u$ (42)

$c_4 + u = 0$ as in Figure (6), fraction (2), order (5)

$\mathfrak{W} = d_3 + u$ (43)

$d_3 + u = 0$ as in Figure (6), fraction (3), order (5)

$\mathfrak{W} = f_2 + u$ (44)

$f_1 + u = 0$ as in Figure (6), fraction (4), order (5)

$\mathfrak{W} = g_1 + u$ (45)

$g_1 + u = 0$ as in Figure (6), fraction (5), order (5)

Mathematically, $\underline{\mathbf{b}}_5$, $\underline{\mathbf{c}}_4$, $\underline{\mathbf{d}}_3$, $\underline{\mathbf{f}}_2$ and $\underline{\mathbf{g}}_1$ are set of elements in mafic magma $\mathcal{M}_m$ such that, $\underline{\mathbf{b}}_5$ is the acidic olivine, $\underline{\mathbf{c}}_4$ is the acidic pyroxene, $\underline{\mathbf{d}}_3$ is the acidic amphibole $\underline{\mathbf{f}}_2$ is the intermediate black mica, $\underline{\mathbf{g}}_1$ is k-feldspar and $\underline{\mathbf{u}}$ is an element in felsic magma $\mathcal{F}_m$ such that $\underline{\mathbf{u}}$ is the acidic plagioclase at 700^0C.

Therefore;

$\mathbf{b_5, c_4, d_3, f_2, g_1} \in \mathcal{M}_m$: b_5 = acidic olivine (Mg<Fe)

:c_3 = acidic pyroxene (Mg<Fe)

:d_2 = acidic amphibole (Mg<Fe)

:f_1 = acidic black mica (Mg<Fe)

$u \in \mathcal{F}_m$**:** u = acidic plagioclase (Ca<Na)

So that**, $\underline{\mathbf{u}}$ maps** $\underline{\mathbf{b}}_5$, $\underline{\mathbf{c}}_4$ $\underline{\mathbf{d}}_3$, $\underline{\mathbf{f}}_2$, $\underline{\mathbf{g}}_1$ in the melt $\mathfrak{W}$ at 700^0C during crystallization of magma under thermodynamic change as gives below;

$\underline{\mathbf{u}} \longrightarrow \underline{\mathbf{b}}_5$, at 700^0C.

$\underline{\mathbf{u}} \longrightarrow \underline{\mathbf{c}}_4$, at 700^0C.

$\underline{\mathbf{u}} \longrightarrow \underline{\mathbf{d}}_3$, at 700^0C.

$\underline{\mathbf{u}} \longrightarrow \underline{\mathbf{f}}_2$, at 700^0C.

$\underline{\mathbf{u}} \longrightarrow \underline{\mathbf{g}}_2$, at 700^0C.

$\mathbf{b_5, c_4, d_3, f_2, g_1}, u \in \mathfrak{W} = 700^0C$

Analogically, one plagioclase feldspar $\underline{\mathbf{u}}$, maps one acidic olivine $\underline{\mathbf{b}}_5$, one acidic pyroxene $\underline{\mathbf{c}}_4$, one acidic amphibole $\underline{d}_3$, one acidic black mica and one k-feldspar in the melt $\mathfrak{W}$ and this mapping is called one to five, mapping in Mathematics.

Therefore from the analogy above, $\underline{\boldsymbol{b}}_5$, $\underline{\mathbf{c}}_4$, $\underline{\mathbf{d}}_3$, $\underline{\mathbf{f}}_2$, $\underline{\mathbf{g}}_1$ and $\underline{\boldsymbol{u}}$ are the fifth six crystal phases to be crystallized from the melt $\mathfrak{W}$ and elected into the Fifth Order of Crystallization in which case $\mathcal{M}_m$ *and* f_m are of different phases of mafic and felsic magmas from which these minerals are differentiated.

Analogy 6: Sixth Order of Crystallization in the Melt $\mathfrak{W}$ under Thermodynamic

As crystallization continues, and progresses to the next sixth order of crystallization under thermodynamic change, the fifth two earlier phases $\underline{\mathbf{u}}$ and $\underline{\mathbf{g}}_1$ of fifth order of crystallization must completely be eliminated from the melt $\mathfrak{W}$, so that the two new formed phases $\underline{\mathbf{w}}$ and $\underline{\mathbf{h}}_1$ with the melt $\mathfrak{W}$, would be at equilibrium at the new temperature of $\mathbf{600^0C}$ which would neutralize the cooling effect of the new formed system composing the crystal phases $\underline{\mathbf{w}}$ and $\underline{\mathbf{h}}_1$ with the melt $\mathfrak{W}$ by releasing more heat to the surrounding, according to Le-chaterllier principle

Given that, the earlier phases of the second order of crystallization is shown in equation (46) below;

$g1+u=0$ (46)

As temperature drops, then the next order of crystallization is shown in equation (47) below;

$g1+w=hn$ (47)

Add equations (46) and (47) to eliminate $\underline{\mathbf{g}}_1$ and $\underline{\mathbf{u}}$**,** from the melt $\mathfrak{W}$ and elect $\underline{\mathbf{w}}$ and $\underline{\mathbf{h}}_n$ into the melt $\mathfrak{W}$ of fourth order of crystallization at $\mathbf{700^0C}$ as given in the simultaneous equation below;

Given that;

$$g1+u=0$$

$$\begin{array}{c} (-)g1+w=\mathrm{h}n \\ \hline u-w=-\mathrm{h}n \end{array}$$

$\underline{\mathbf{u}}$ and $\underline{\mathbf{g}}_1$ can be completely eliminated from the melt $\mathfrak{W}$, so that $\underline{\mathbf{w}}$ and $\underline{\mathbf{h}}_n$ can be elected into the melt $\mathfrak{W}$ of fifth order of crystallization which is neutralized by the cooling effect at $\mathbf{600^0C}$ and remain at equilibrium with the melt $\mathfrak{W}$ at $\mathbf{600^0C}$

$u = -h_n + w$ (48)

From equation (40) of the fifth order;

$\mathfrak{W} = \mathbf{b_n - c_n - d_n - f_n - g_n + u}$ (49)

n = 6, for acidic olivine

n = 5, acidic pyroxene

n = 4, for acidic amphibole

n = 3, for intermediate black mica

n = 2, for k-feldspar

$\mathfrak{W} = b_6 - c_5 - d_4 - f_3 - g_2 + u$ (50)

Put the value of $\underline{\mathbf{u}}$ in (71), so that;

$\mathfrak{W} = \mathbf{b_6 - c_5 - d_4 - f_3 - g_2 - h_n + w}$ (51)

Therefore;

$\mathbf{b_6 - c_5 - d_4 - f_3 - g_2} - h_n + w$ is the Sixth Order of Crystallization in the melt $\mathfrak{W}$ under thermodynamic condition.

n = 1, white mica, then;

$\mathfrak{W} = b_6 - c_5 - d_4 - f_3 - g_2 - h_1 + w$

$\mathfrak{W} = b_6 - c_5 - d_4 - f_3 - g_2 - h_1 + w$ (52)

Since $\underline{\mathbf{b}}_6$, $\underline{\mathbf{c}}_5$, $\underline{\mathbf{d}}_4$, $\underline{\mathbf{f}}_3$, $\underline{\mathbf{g}}_2$, $\underline{\mathbf{h}}_1$, and $\underline{\mathbf{w}}$ are in equilibrium with the melt $\mathfrak{W}$ and the negative sign indicates that both the $\underline{\mathbf{b}}_6$, $\underline{\mathbf{c}}_5$, $\underline{\mathbf{d}}_4$, $\underline{\mathbf{f}}_3$, $\underline{\mathbf{g}}_2$ and $\underline{\mathbf{h}}_1$ are of different fractions while $\underline{\mathbf{w}}$ is distributed across all fractions of $\underline{\mathbf{b}}_6$, $\underline{\mathbf{c}}_5$, $\underline{\mathbf{d}}_4$, $\underline{\mathbf{f}}_3$, $\underline{\mathbf{g}}_2$ and $\underline{\mathbf{h}}_1$, therefore;

$\underline{\mathbf{b}}_6$, $\underline{\mathbf{c}}_5$, $\underline{d}_4$, $\underline{\mathbf{f}}_3$, $\underline{\mathbf{g}}_2$, $\underline{\mathbf{h}}_1 \in \mathcal{M}_m$ and $\underline{\mathbf{w}}$, $\in \mathcal{F}_m$

$\mathcal{M}_m + \mathcal{F}_m = 0$

$\mathcal{M}_m = b_6 - c_5 - d_4 - f_3 - g_2 - h_1$

$\mathcal{F}_m = w$

$\mathfrak{W} = b_6 + w$ (53)

$b_6 + y = 0$ as in Figure (6), fraction (1) and order (6)

$\mathfrak{W} = c_5 + w$ (54)

$c_5 + y = 0$ as in Figure (6), fraction (2) and order (6)

$\mathfrak{W} = d_4 + w$ (55)

$d_4 + y = 0$ as in Figure (6), fraction (3) and order (6)

$\mathfrak{W} = f_3 + w$ (56)

$F_3 + y = 0$ as in Figure (6), fraction (4) and order (6)

$\mathfrak{W} = g_2 + w$ (57)

$g_2 + y = 0$ as in Figure (6), fraction (5) and order (6)

$\mathfrak{W} = h_1 + w$ (58)

$h_1 + w = 0$ as in Figure (6), fraction (6) and order (6)

Mathematically, $\underline{\mathbf{b}}_6$, $\underline{\mathbf{c}}_5$, $\underline{\mathbf{d}}_4$, $\underline{\mathbf{f}}_3$, and $\underline{\mathbf{g}}_2$, are set of elements in mafic magma $\mathcal{M}_m$ such that, $\underline{\mathbf{b}}_6$ is the acidic olivine, $\underline{\mathbf{c}}_5$ is the acidic pyroxene, $\underline{\mathbf{d}}_4$ is the acidic amphibole $\underline{\mathbf{f}}_3$ is the intermediate black mica, $\underline{\mathbf{g}}_2$ is k-feldspar and $\underline{\mathbf{h}}_1$ is the acidic white mica and $\underline{\mathbf{w}}$ is an element in felsic magma $\mathcal{F}_m$ such that $\underline{\mathbf{w}}$ is the acidic plagioclase at 600^0C.

Therefore;

$\mathbf{b_6, c_5, d_4, f_3, g_2, h_1} \in \mathcal{M}_m$: b_6 = acidic olivine (Mg<Fe)

:c_5 = acidic pyroxene (Mg<Fe)

:d_4 = acidic amphibole (Mg<Fe)

:f_3 = acidic black mica (Mg<Fe)

:g_2 = acidic-feldspar (Mg<Fe)

:h_1 = acidic white mica (Mg<Fe)

$\mathbf{w} \in Ƒ_m$**:** w = acidic plagioclase(Ca<Na)

So that**, $\underline{\mathbf{w}}$ maps** $\underline{\mathbf{b}}_6$, $\underline{\mathbf{c}}_5$ $\underline{\mathbf{d}}_4$, $\underline{\mathbf{f}}_3$, $\underline{\mathbf{g}}_2$ and $\underline{\mathbf{h}}_1$ in the melt $\mathfrak{W}$ at 700^{0}C during crystallization of magma under thermodynamic change as giving below;

$\underline{\mathbf{w}} \longrightarrow \underline{\mathbf{b}}_6$, at 600^{0}C.

$\underline{\mathbf{w}} \longrightarrow \underline{\mathbf{c}}_5$, at 600^{0}C.

$\underline{\mathbf{w}} \longrightarrow \underline{\mathbf{d}}_4$, at 600^{0}C.

$\underline{\mathbf{w}} \longrightarrow \underline{\mathbf{f}}_3$, at 600^{0}C.

$\underline{\mathbf{w}} \longrightarrow \underline{\mathbf{g}}_2$, at 600^{0}C.

$\underline{\mathbf{w}} \longrightarrow \underline{\mathbf{h}}_1$, at 600^{0}C

$\mathbf{b}_6$, $\mathbf{b}_5$, $\mathbf{c}_4$, $\mathbf{d}_3$, $\mathbf{f}_2$, $\mathbf{g}_1$, $\mathbf{w} \in \mathfrak{W}$ = 600^{0}C

Analogically, one plagioclase feldspar $\underline{\mathbf{w}}$, maps one acidic olivine $\underline{\mathbf{b}}_6$, one acidic pyroxene $\underline{\mathbf{c}}_5$, one acidic amphibole $\underline{d}_4$, one acidic black mica, one sodic-feldspar $\underline{\mathbf{g}}_2$ and one white mica $\underline{\mathbf{h}}_1$ in the melt $\mathfrak{W}$ and this mapping is called one to six, mapping in Mathematics.

Therefore from the analogy above, $\underline{\boldsymbol{b}}_6$, $\underline{\mathbf{c}}_5$, $\underline{\mathbf{d}}_4$, $\underline{\mathbf{f}}_3$, $\underline{\mathbf{g}}_2$, $\underline{\mathbf{h}}_1$ and $\underline{\boldsymbol{w}}$ are the sixth, six crystal phases to be crystallized from the melt $\mathfrak{W}$ and elected into the Sixth Order of Crystallization in which case $\mathscr{M}_m$ and f_m are of different phases of mafic and felsic magmas from which these minerals are differentiated.

Analogy 7: Seventh Order of Crystallization in the Melt $\mathfrak{W}$ under Thermodynamic

As crystallization continues, and progresses to the next sixth order of crystallization under thermodynamic change, the fifth two earlier phases $\underline{\mathbf{w}}$ and $\underline{\mathbf{h}}_1$ of fifth order of crystallization must completely be eliminated from the melt $\mathfrak{W}$, so that the two new formed phases $\underline{\mathbf{y}}$ and $\underline{\mathbf{k}}_1$ with the melt $\mathfrak{W}$, would be at equilibrium at the new temperature of **600^{0}C** which would neutralize the cooling effect of the new formed system composing the crystal phases $\underline{\mathbf{w}}$ and $\underline{\mathbf{k}}_1$ with the melt $\mathfrak{W}$ by releasing more heat to the surrounding, according to Le-chaterllier principle

Given that, the earlier phases of the second order of crystallization is shown in equation (80) below;

$$h1+w=0 \qquad (59)$$

As temperature drops, then the next order of crystallization is shown in equation (60) below;

$$h1+y=kn \qquad (60)$$

Add equations (59) and (60) to eliminate $\underline{\mathbf{h}}_1$ and $\underline{\mathbf{w}}$**,** from the melt $\mathfrak{W}$ and elect $\underline{\mathbf{y}}$ and $\underline{\mathbf{k}}_n$ into the melt $\mathfrak{W}$ of fourth order of crystallization at **500^{0}C** as given in the simultaneous equation below;

Given that;

$$\begin{array}{l} h1+w=0 \\ \\ \underline{(-)h1+y=kn} \\ w-y=-kn \end{array}$$

$\underline{\mathbf{w}}$ and $\underline{\mathbf{h}}_1$ can be completely eliminated from the melt $\mathfrak{W}$, so that $\underline{\mathbf{y}}$ and $\underline{\mathbf{k}}_n$ can be elected into the melt $\mathfrak{W}$ of fifth order of crystallization which is neutralized by the cooling effect at **500^{0}C** and remain at equilibrium with the melt $\mathfrak{W}$ at **500^{0}C**

$$w = -k_n + y \qquad (61)$$

From equation (72) of the sixth order;

$$\mathfrak{W} = \mathbf{b_n - c_n - d_n - f_n - g_n - h_n + w} \qquad (62)$$

n = 7 for acidic olivine

n = 6 for acidic pyroxene

n = 5 for acidic amphibole

n = 4 for intermediate black mica

n = 3 for sodic- k-feldspar

n = 2 for acidic white mica

$\mathfrak{W} = b_7 - c_6 - d_5 - f_4 - g_3 - h_2 + w$ (63)

Put the value of **w** in (83), so that;

$\mathfrak{W} = \mathbf{b_7 - c_6 - d_5 - f_4 - g_3 - h_2 - k_n} + \mathbf{y}$ (64)

Therefore;

$\mathbf{b_7 - c_6 - d_5 - f_4 - g_3 - h_2 - k_n} + \mathbf{y}$ is the Seventh Order of Crystallization in the melt $\mathfrak{W}$ under thermodynamic condition.

n = 1, for acidic silica;

$\mathfrak{W} = b_7 - c_6 - d_5 - f_4 - g_3 - h_2 - k_1 + w$

$\mathfrak{W} = b_7 - c_6 - d_5 - f_4 - g_3 - h_2 - k_1 + w$ (65)

Since $\underline{\mathbf{b}}_7$, $\underline{\mathbf{c}}_6$, $\underline{\mathbf{d}}_5$, $\underline{\mathbf{f}}_4$, $\underline{\mathbf{g}}_3$, $\underline{\mathbf{h}}_2$, $\underline{\mathbf{k}}_1$ and $\underline{\mathbf{y}}$, are in equilibrium with the melt $\mathfrak{W}$ and the negative sign indicates that both the $\underline{\mathbf{b}}_7$, $\underline{\mathbf{c}}_6$, $\underline{\mathbf{d}}_5$, $\underline{\mathbf{f}}_4$, $\mathbf{g}_3$, $\underline{\mathbf{h}}_2$ and $\underline{\mathbf{k}}_1$ are of different fractions while $\underline{\mathbf{y}}$ is distributed across all fractions of $\underline{\mathbf{b}}_7$, $\underline{\mathbf{c}}_6$, $\underline{\mathbf{d}}_5$, $\underline{\mathbf{f}}_4$, $\mathbf{g}_3$, $\underline{\mathbf{h}}_1$, and $\underline{\mathbf{k}}_1$, therefore;

$\underline{\mathbf{b}}_7, \underline{\mathbf{c}}_6, \underline{\mathbf{d}}_5, \underline{\mathbf{f}}_4, \underline{\mathbf{g}}_3, \underline{\mathbf{h}}_2, \underline{\mathbf{k}}_1 \in \mathcal{M}_m \;\mathbf{\Lambda}\; \underline{\mathbf{y}} \in Ƒ_m$

$\mathcal{M}_m + Ƒ_m = 0$

$\mathcal{M}_m = b_7 - c_6 - d_5 - f_4 - g_3 - h_2 - k_1$

$Ƒ_m = y$

$\mathfrak{W} = b_7 + y$ (66)

$\mathbf{b_7 + w = 0}$ as in Figure (6), fraction (1), order (7)

$\mathfrak{W} = c_6 + y$ (67)

$\mathbf{c_6 + w = 0}$ as in Figure (6), fraction (2), order (7)

$\mathfrak{W} = d_5 + y$ (68)

$\mathbf{d_5 + w = 0}$ as in Figure (6), fraction (3), order (7)

$\mathfrak{W} = f_4 + y$ (69)

$\mathbf{f_4 + w = 0}$ as in Figure (6), fraction (4), order (7)

$\mathfrak{W} = g_3 + y$ (70)

$\mathbf{g_3 + w = 0}$ as in Figure (6), fraction (5), order (7)

$\mathfrak{W} = h_2 + y$ (71)

$\mathbf{h_2 + y = 0}$ as in Figure (6), fraction (6), order (7)

$\mathfrak{W} = k_1 + y$ (72)

$\mathbf{k_1 + y = 0}$ as in Figure (6), fraction (6), order (7)

Mathematically, $\underline{\mathbf{b}}_7$, $\underline{\mathbf{c}}_6$, $\underline{\mathbf{d}}_5$, $\underline{\mathbf{f}}_4$, $\underline{\mathbf{g}}_3$, $\underline{\mathbf{h}}_1$ and $\underline{\mathbf{k}}_1$ are set of elements in mafic magma $\mathcal{M}_m$ such that, $\underline{\mathbf{b}}_7$ is the acidic olivine, $\underline{\mathbf{c}}_6$ is the acidic pyroxene, $\underline{\mathbf{d}}_5$ is the acidic amphibole $\underline{\mathbf{f}}_4$ is the acidic black mica, $\underline{\mathbf{g}}_3$ is sodic-k-feldspar, $\underline{\mathbf{h}}_1$ is the acidic white mica, $\underline{\mathbf{k}}_1$ is the acidic silica and $\underline{\mathbf{y}}$ is an element in felsic magma $Ƒ_m$ such that $\underline{\mathbf{y}}$ is the fluid phase containing incompatible elements at 500^0C.

Therefore;

$\mathbf{b_7, c_6, d_5, f_4, g_3, h_2, k_1} \in \mathcal{M}_m$: b_7 = acidic olivine (Mg<Fe)

: c_6 = acidic pyroxene (Mg<Fe)

: d_5 = acidic amphibole (Mg<Fe)

: f_4 = acidic black mica (Mg<Fe)

: g_3 = sodic-feldspar (K=Na)

: h_2 = acidic white mica (K>Na)

: k_1 = acidic silica (k<Na)

$y \in \mathcal{F}_m$**:** y = fluid phase containing incompatible trace elements

So that**, $\underline{\mathbf{y}}$ maps** $\underline{\mathbf{b}}_7$, $\underline{\mathbf{c}}_6$ $\underline{\mathbf{d}}_5$, $\underline{\mathbf{f}}_4$, $\underline{\mathbf{g}}_3$, $\underline{\mathbf{h}}_2$ and $\underline{\mathbf{k}}_1$ in the melt $\mathfrak{W}$ at 500^0C during crystallization of magma under thermodynamic change as gives below;

$\underline{\mathbf{y}} \longrightarrow \underline{\mathbf{b}}_7$, at 500^0C.

$\underline{\mathbf{y}} \longrightarrow \underline{\mathbf{c}}_6$, at 500^0C.

$\underline{\mathbf{y}} \longrightarrow \underline{\mathbf{d}}_5$, at 500^0C.

$\underline{\mathbf{y}} \longrightarrow \underline{\mathbf{f}}_4$, at 500^0C.

$\underline{\mathbf{y}} \longrightarrow \underline{\mathbf{g}}_3$, at 500^0C.

$\underline{\mathbf{y}} \longrightarrow \underline{\mathbf{h}}_2$, at 500^0C

$\underline{\mathbf{y}} \longrightarrow \underline{\mathbf{k}}_1$, at 500^0C

$\mathbf{b}_7, \mathbf{c}_6, \mathbf{d}_5, \mathbf{f}_4, \mathbf{g}_3, \mathbf{h}_2, \mathbf{k}_1, y \in \mathfrak{W} = 500^0$C

Analogically, fluid $\underline{\mathbf{y}}$, maps one acidic olivine $\underline{\mathbf{b}}_7$, one acidic pyroxene $\underline{\mathbf{c}}_6$, one acidic amphibole $\underline{d}_5$, one acidic black mica $\underline{\mathbf{f}}_4$, one sodic-feldspar $\underline{\mathbf{g}}_3$, one white mica $\underline{\mathbf{h}}_2$ and one acidic silica $\underline{\mathbf{k}}_1$ in the melt $\mathfrak{W}$ and this mapping is called one to seven, mapping in Mathematics.

Therefore from the analogy above, $\underline{b}_7$, $\underline{c}_6$, $\underline{d}_5$, $\underline{f}_4$, $\underline{g}_3$, $\underline{h}_2$, $\underline{k}_1$, are the seventh, seven crystal phases and $\underline{\mathbf{y}}$, the fluid phase to be crystallized from the melt $\mathfrak{W}$ and elected into the Seventh Order of Crystallization in which case $\mathcal{M}_m$ and f_m are of different phases of mafic and felsic magmas from which these minerals are differentiated as shown Figure (6).

In conclusion, $\underline{\mathbf{b}}_7$, $\underline{\mathbf{c}}_6$, $\underline{\mathbf{d}}_5$, $\underline{\mathbf{f}}_4$, $\underline{\mathbf{g}}_3$, $\underline{\mathbf{h}}_2$, $\underline{\mathbf{k}}_1$ are in equilibrium with fluid phase $\underline{\mathbf{y}}$, at 500^0C. $\underline{\mathbf{y}}$, is the remaining and the last fluid which hosts other elements which cannot partitioned along with crystal phases but concentrated in the fluid $\underline{\mathbf{y}}$ and these elements are called Imcompatible Trace Element.

From the analyses above, **b, c, d, f, g, h, k** and **a, e, i, o, u, w, y**, can be mapped as;

$(\mathbf{b}_n + \mathbf{a}), (\mathbf{c}_n + \mathbf{e}), (\mathbf{d}_n + \mathbf{i}), (\mathbf{g}_n + \mathbf{o}), (\mathbf{h}_n + \mathbf{w}), (\mathbf{k}_n + \mathbf{y})$ which can be tabulated in the following Tables 4, 5 and can be analyzed using Figure 6, below;

Where **b, c, d, f, g, h**, and **k** are Mafic minerals and **a, e, i, o, u, y**, and **w**, are Felsic minerals and can be represented in order of crystallization as shown below;

4.3 Summary of Order of Crystallization from the Melt, $\mathfrak{W}$

- $\mathbf{b}_n + \mathbf{a}$**:** *First Order of Crystallization*
- $\mathbf{b}_2 - \mathbf{c}_n + \mathbf{e}$**:** *Second Order of Crystallization*
- $\mathbf{b}_3 - \mathbf{c}_2 - \mathbf{d}_n + \mathbf{i}$**:** *Third Order of Crystallization*
- $\mathbf{b}_4 - \mathbf{c}_3 - \mathbf{d}_2 - \mathbf{f}_n + \mathbf{o}$**:** *Fourth Order of Crystallization*
- $\mathbf{b}_5 - \mathbf{c}_4 - \mathbf{d}_3 - \mathbf{f}_2 - \mathbf{g}_n + \mathbf{u}$**:** Fifth Order of Crystallization
- $\mathbf{b}_6 - \mathbf{c}_5 - \mathbf{d}_4 - \mathbf{f}_3 - \mathbf{g}_2 - \mathbf{h}_n + \mathbf{w}$**:** *Sixth Order of Crystallization*
- $\mathbf{b}_7 - \mathbf{c}_6 - \mathbf{d}_5 - \mathbf{f}_4 - \mathbf{g}_3 - \mathbf{h}_2 - \mathbf{k}_n + \mathbf{y}$**:** *Seventh Order of Crystallization*
- $n \in \mathcal{M}_m$ **: n = 1: Polymerization series.**

That is, for all polymerization series, n = 1

- $n, p \in \mathcal{M}_m : n = n \longrightarrow p$**: Isomorphous series.**

That is, for all isomorphous series, $n = 1 \longrightarrow p$

$\mathcal{M}_m$ = mafic magma

n = number of times basic minerals can form the mafic melt, $\mathcal{M}_m$.

P = number of times acidic minerals can form from the mafic melt, $\mathcal{M}_m$.

Therefore from the beginning to the end of crystallization under thermodynamic change, "*the magma* $\mathfrak{W}$ ": changes from the basic magma β_m to acidic magma α_m in mafic type magma $\mathcal{M}_m$, and basic magma β_m to alkali magma α_m in felsic type magma $\mathcal{F}_m$, hence four different types of magma were mapped out from the magma $\mathfrak{W}$, at different temperatures as shown in Table (1)

Above and Concept 1 to 7 below and these magmas are;

- Basic magma:40-49% silica
- Acidic magma: 65-75%
- Alkali magma: 65-75% silica
- Intermediate magm:50-64% silica

Therefore;

$\mathcal{M}_m = \beta_m + \alpha_m : \beta_m$ = Basic magma

$: \alpha_m$ = Acidic magma

Ƒm = $\beta_m + \alpha_m : \beta_m$ = Basic magma

$: \alpha_m$ = Alkali magma

: Ƒm = Felsic magma

Therefore;

basic magma (β_m), acidic magma (α_m) are identity elements ($\in$) in mafic magma () and (Λ) basic magma β_m , alkali magma (α_m) are identity elements ($\in$) in felsic magma Ƒm such that(**:**) , mafic magma (), felsic magma (Ƒm) are identity elements ($\in$) in the recharged magma $\mathfrak{W}$ and this is mathematically represented below;

$\beta_m , \alpha_m \in \mathbf{\Lambda} \beta_m , \alpha_m \in$ Ƒm**:** , Ƒm $\in \mathfrak{W} \ \mathbf{\Lambda} \mathfrak{W} \in \mathfrak{R}$

$\mathfrak{W}$ = recharged magma

$\mathfrak{R}$ = resident magma

- Recharged magma $\mathfrak{W}$ is the magma that is partially electrolytic under thermodynamic change which allows the movement of electrons in an orderly manner. This happens on only when there is thermodynamic change in the resident magma $\mathfrak{R}$
- Resident magma $\mathfrak{R}$ is the magma in which electrons move in a chaotic manner at high temperature without any thermodynamic change.

4.4 The New concept of Order of Crystallization using the two Reactions Series, *Polymerization* and *Isomorphous* Reactions Series

The crystallization by elections of materials into the melt $\mathfrak{W}$ and eliminations of materials from the melt $\mathfrak{W}$, from the beginning to the end of crystallization with respect to Gibbs free energy at 1atm are presented in Tables 1 and 2 using Bowen's and Goldschmidt Concepts and Bowen's Concepts.

Table 1. Mapping of Magma using Minerals and Rocks in Bowen's Concept

MAFIC MINERAL		**FELSIC MINERAL**		**ROCK TYPE**	**TEMPERATURE (^{0}C)**
Consonants	Minerals	Vowels	Minerals	Rocks	
b_n	Olivine	A	Anorthite	Perridotite	2000
c_n	Pyroxene	E	Bytownite	Basalt	1000
d_n	Amphibole	I	Labr adorite	Basalt	900
f_n	Biotite	O	Andesine	Andesite	800
g_n	Muscovite	U	Oligoclase	Dacite	700
h_n	Orthoclase	W	Albite	Rhyolite	600
k_n	Quartz	Y	Quartz	Quartz	500
Z	**HYDROTHERMAL FLUID**				400

Table 2. Order of Crystallization of Magma using Goldschmidt Concept

Order of Crystallization	FELSIC	MAFIC							
	$\mathcal{F}_m$	$\mathcal{M}_m$	b_{n+p}	c_{n+p}	d_{n+p}	f_{n+p}	g_{n+p}	h_{n+p}	k_{n+p}
		Index number (n) $n = 1 \longrightarrow 7$	1	2	3	4	5	6	7
First	A	1	b_1	c_0	d_0	f_0	g_0	h_0	k_0
Second	E	2	b_2	c_1	d_0	f_0	g_0	h_0	k_0
Third	I	3	b_3	c_2	d_1	f_0	g_0	h_0	k_0
Fourth	O	4	b_4	c_3	d_2	f_1	g_0	h_0	k_0
Fifth	U	5	b_5	c_4	d_3	f_2	g_1	h_0	k_0
Six	W	6	b_6	c_5	d_4	f_3	g_2	h_1	k_0
Seventh	Y	7	b_7	c_6	d_5	f_4	g_3	h_2	k_1

A. Concepts 1

- For polymerization process, $b_n \longrightarrow k_n$, as $n = 1$, as shown in concept 1 and Table (1) above. Therefore for polymerization reaction 'n' has specific value from the beginning to the end of crystallization. Therefore, 'n' = 1, is the only one root of the homogeneous formed crystal, hence strongly stable according to Adams and Bashforth method.

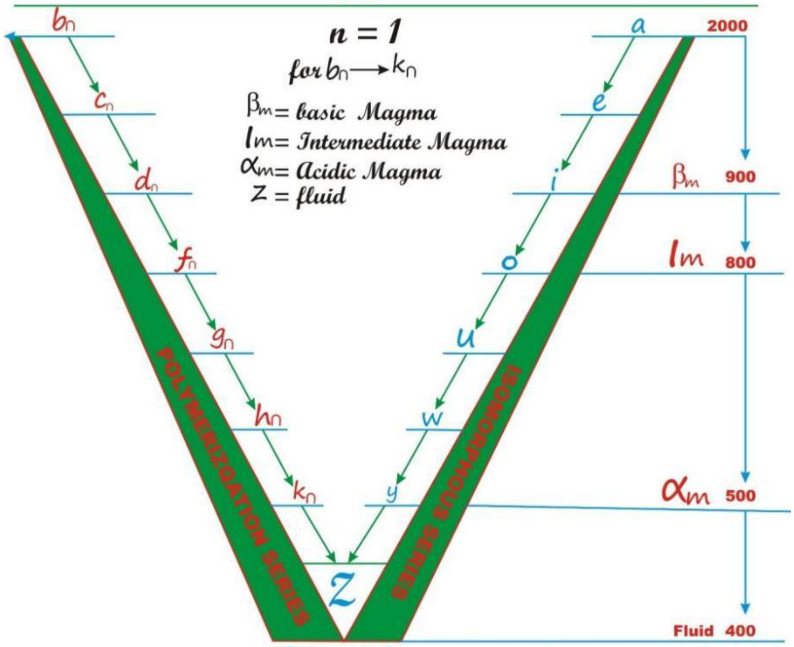
- For isomorphous process and for a given specific value of 'n', there is always a unique value of 'p' as shown in Table (2) above.

Therefore for, $b_n \longrightarrow b_p$, as $n = 1$ and $p = 7$

Then, $b_n \longrightarrow b_p$; $= b_1 \longrightarrow b_7$; this means that there is continuous change of n to p during crystallization by ionic substitution of 'n' by 'p' in solid solution as given in concept **A1** and concept **A2.**

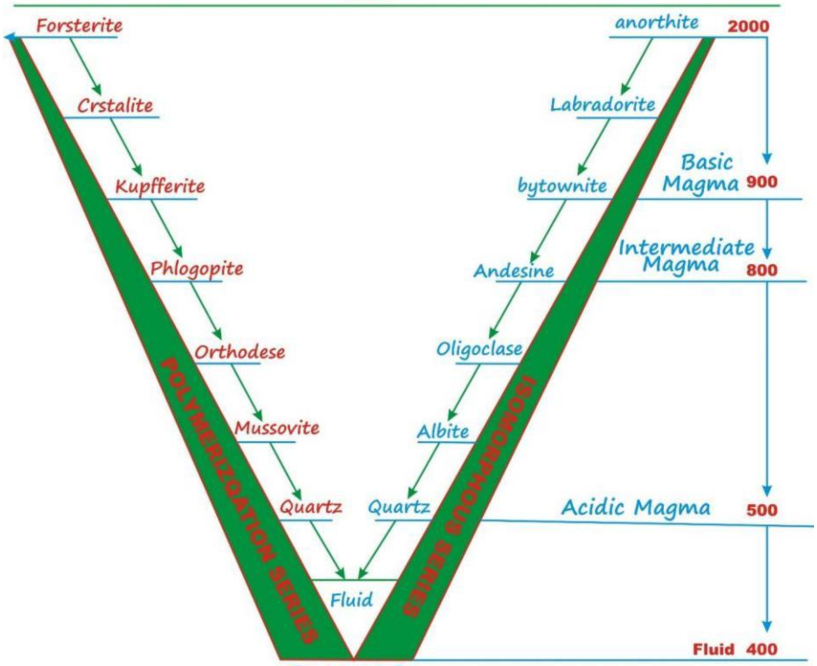
Therefore, 'p' and 'n' are the two roots of the heterogeneous formed crystal, hence weakly stable. The root of the heterogeneous crystal, n:p = (1, 7) as given in Concept 1, below.

A1



Concept 1

A2



Concept 2

B. Concept 2

- For polymerization process, $b_n \longrightarrow h_n$, as n = 2, as shown in concept 2 and Table (1) above. Therefore for polymerization reaction 'n' has specific value from the beginning to the end of crystallization. Therefore, 'n' = 2, is the only one root of the homogeneous formed crystal, hence strongly stable according to Adams and Bash.

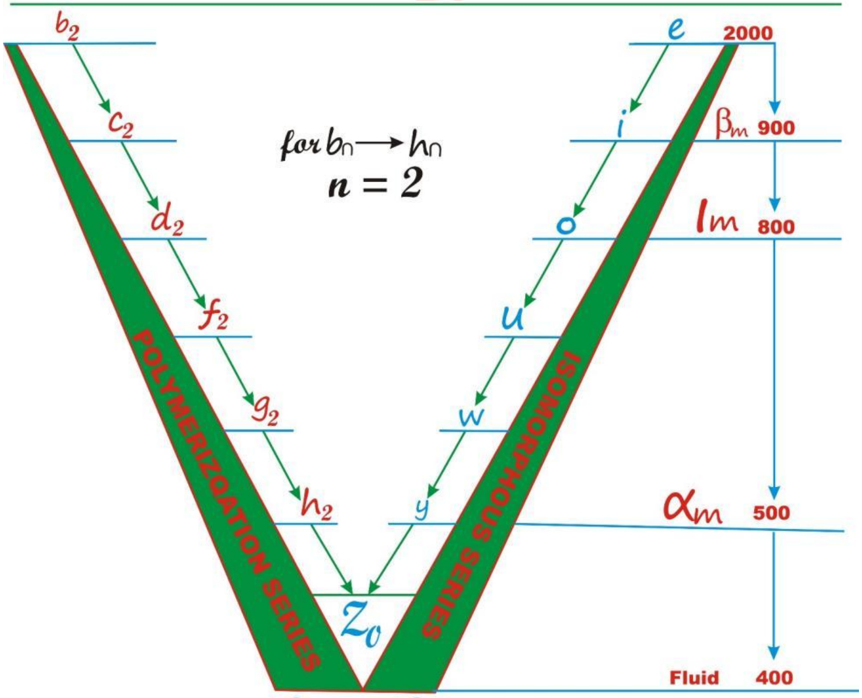
- For isomorphous process and for a given specific value of 'n', there is always a unique value of 'p' as shown in Table (2) above, Concept 2.

Therefore for, $b_n \longrightarrow b_p$, as n = 2 and p = 6

Then, $b_n \longrightarrow b_p$; = $b_2 \longrightarrow b_6$; this means that there is continuous change of n to p during crystallization by ionic substitution of 'n' by 'p' in solid solution as given in concept **B1** and concept **B2** below**.**

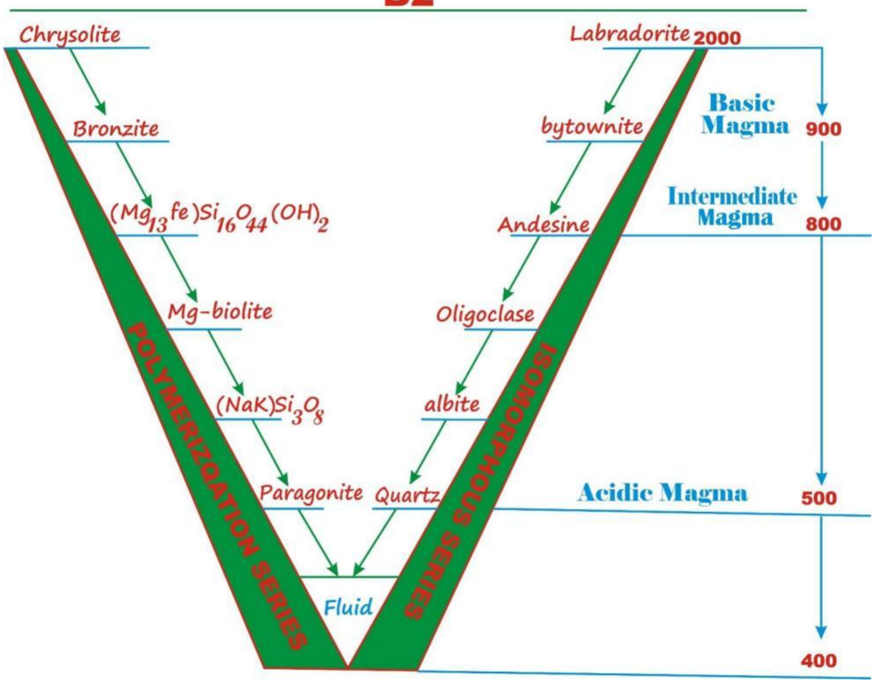
Therefore, 'p' and 'n' are the two roots of the heterogeneous formed crystal, hence weakly stable. The roots of the heterogeneous crystal, n:p = (2, 6).

B1



Concept 2

B2



C. Concept 3

- For polymerization process, $d_n \longrightarrow g_n$, as n = 3, as shown in concept 3 and Table (1) above. Therefore for polymerization reaction ‘n’ has specific value from the beginning to the end of crystallization. Therefore, ‘n’ = 3, is the only one root of the homogeneous formed crystal, hence strongly stable according to Adams and Bash.

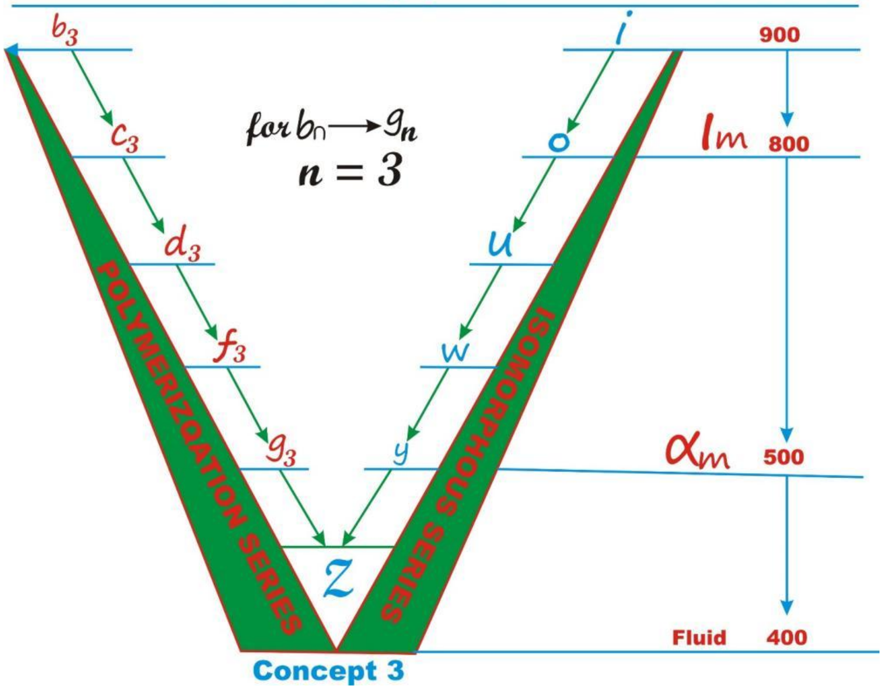
- For isomorphous process and for a given specific value of ‘n’, there is always a unique value of ‘p’ as shown in Table (2) above, Concept 3.

Therefore for, $b_n \longrightarrow b_p$, as n = 3 and p = 5

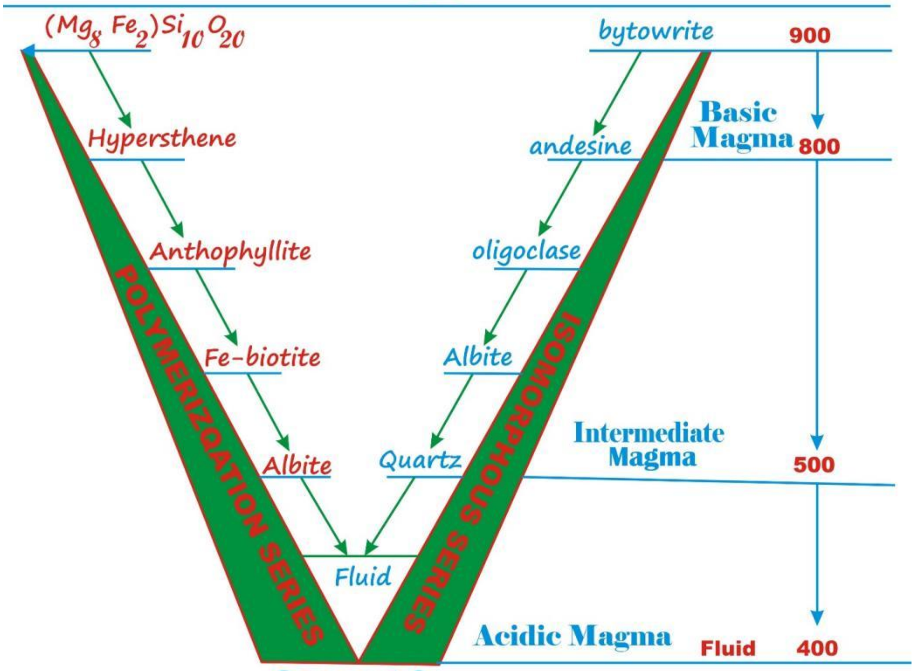
Then, $b_n \longrightarrow b_p$; = $b_3 \longrightarrow b_5$; this means that there is continuous change of n to p during crystallization by ionic substitution of ‘n’ by ‘p’ in solid solution as given in concept **C1** and concept **C2** below**.**

Therefore, ‘p’ and ‘n’ are the two roots of the heterogeneous formed crystal, hence weakly stable. The roots of the heterogeneous crystal, n:p = (3, 5).

C1



C2



Concept 3

D. Concept 4

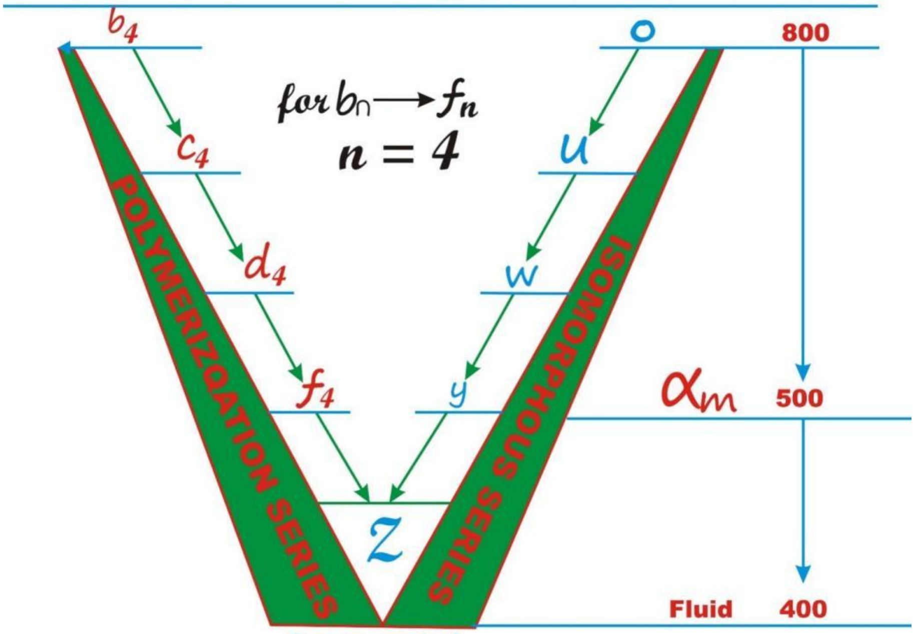
- For polymerization process, $b_n \longrightarrow f_n$, as n = 4, as shown in concept 4 and Table (1) above. Therefore for polymerization reaction 'n' has specific value from the beginning to the end of crystallization. Therefore, 'n' = 4, is the only one root of the homogeneous formed crystal, hence strongly stable according to Adams and Bash.
- For isomorphous process and for a given specific value of 'n', there is always a unique value of 'p' as shown in Table (2) above, Concept 4.

Therefore for, $b_n \longrightarrow f_p$, as n = 4 and p = 4

Then, $b_n \longrightarrow b_p$; $= b_4 \longrightarrow b_4$; this means that there is continuous change of n to p during crystallization by ionic substitution of 'n' by 'p' in solid solution as given in concept **D1** and concept **D2** below**.**.

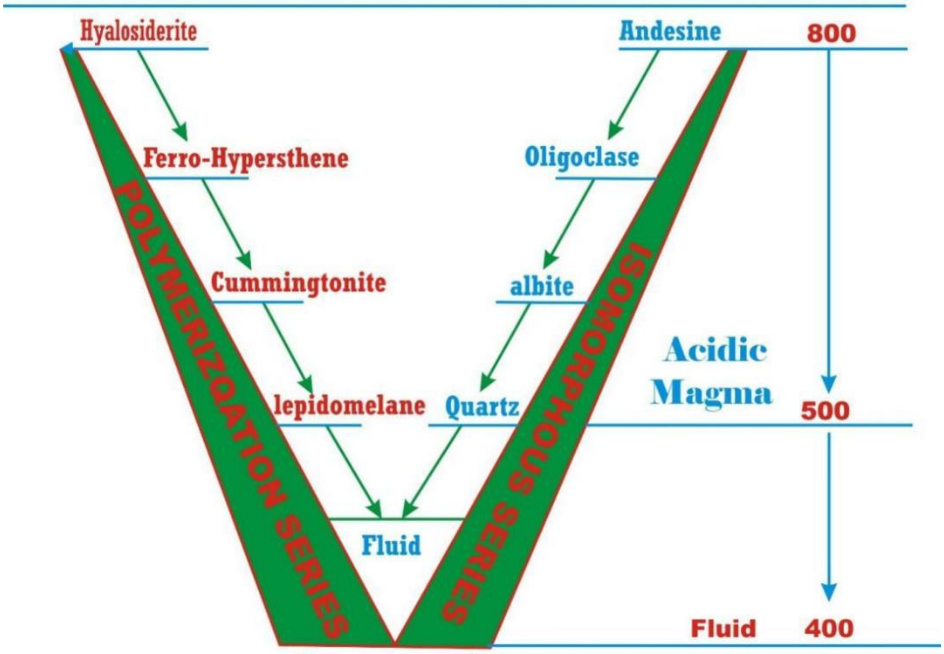
Therefore, 'p' and 'n' are the two roots of the heterogeneous formed crystal, hence weakly stable, therefore the roots of the heterogeneous crystal, n: p = (4, 4).

D1



Concept 4

D2



Concept 4

E. Concept 5

- For polymerization process, $b_n \longrightarrow d_n$, as n = 5, as shown in concept 5 and Table (1), above. Therefore for polymerization reaction 'n' has specific value from the beginning to the end of crystallization. Therefore, 'n' = 5, is the only one root of the homogeneous formed crystal, hence strongly stable according to Adams and Bash.

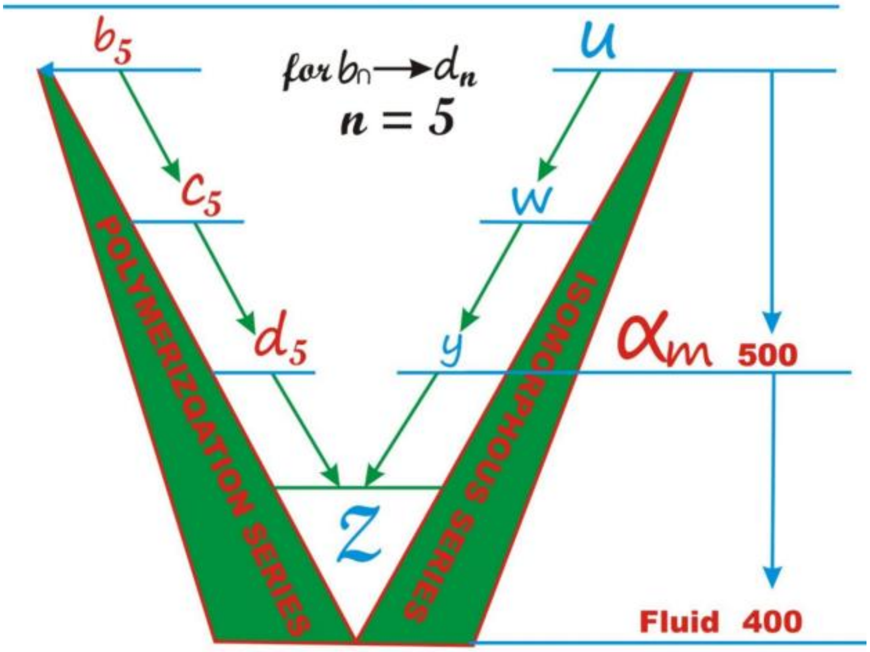
- For isomorphous process and for a given specific value of 'n', there is always a unique value of 'p' as shown in Table (2) above, Concept 5.

Therefore for, $b_3 \longrightarrow b_5$, as n = 5 and p = 3

Then, $b_n \longrightarrow b_p$; $= b_5 \longrightarrow b_3$; this means that there is continuous change of n to p during crystallization by ionic substitution of 'n' by 'p' in solid solution as given in concept **E1** and concept **E2** below.

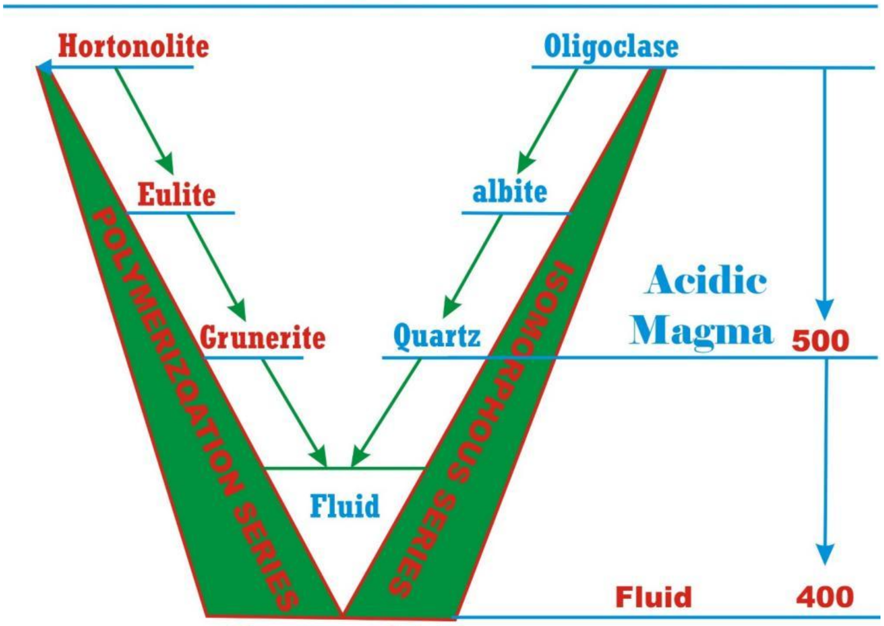
Therefore, 'p' and 'n' are the two roots of the heterogeneous formed crystal, hence weakly stable, therefore the roots of the heterogeneous crystal, n: p = (5, 3).

E1



Concept 5

E2



Concept 5

F. Concept 6

- For polymerization process, $b_n \longrightarrow c_n$, as n = 6, as shown in concept 2 and Table (1) above. Therefore for polymerization reaction ‘n’ has specific value from the beginning to the end of crystallization. Therefore, ‘n’ = 6, is the only one root of the homogeneous formed crystal, hence strongly stable according to Adams and Bash.

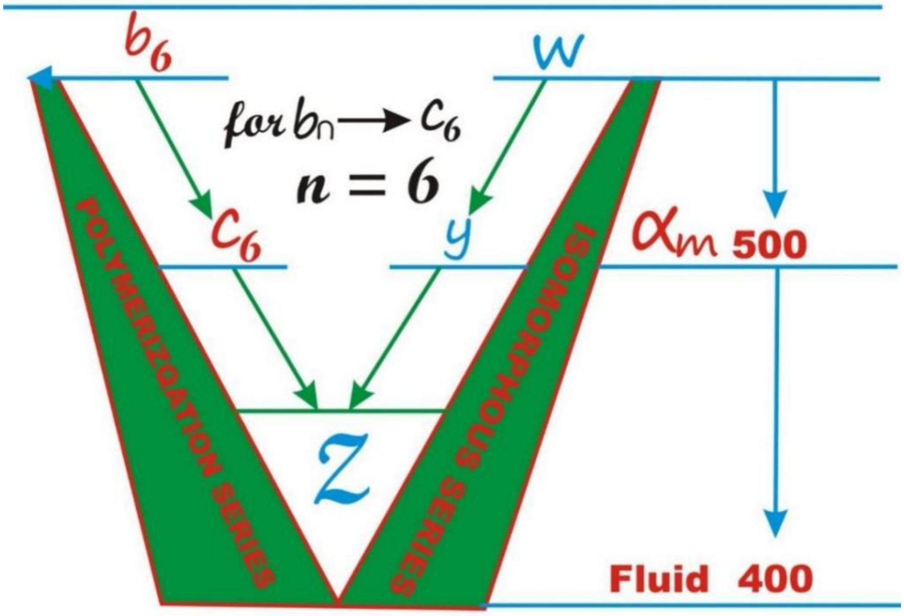
- For isomorphous process and for a given specific value of ‘n’, there is always a unique value of ‘p’ as shown in Table (2) above, Concept 6.

Therefore for, $b_2 \longrightarrow b_6$, as n = 6 and p = 2

Then, $b_n \longrightarrow b_p$; = $b_6 \longrightarrow b_2$; this means that there is continuous change of n to p during crystallization by ionic substitution of ‘n’ by ‘p’ in solid solution as given in concept **F1** and concept **F2** below**.**

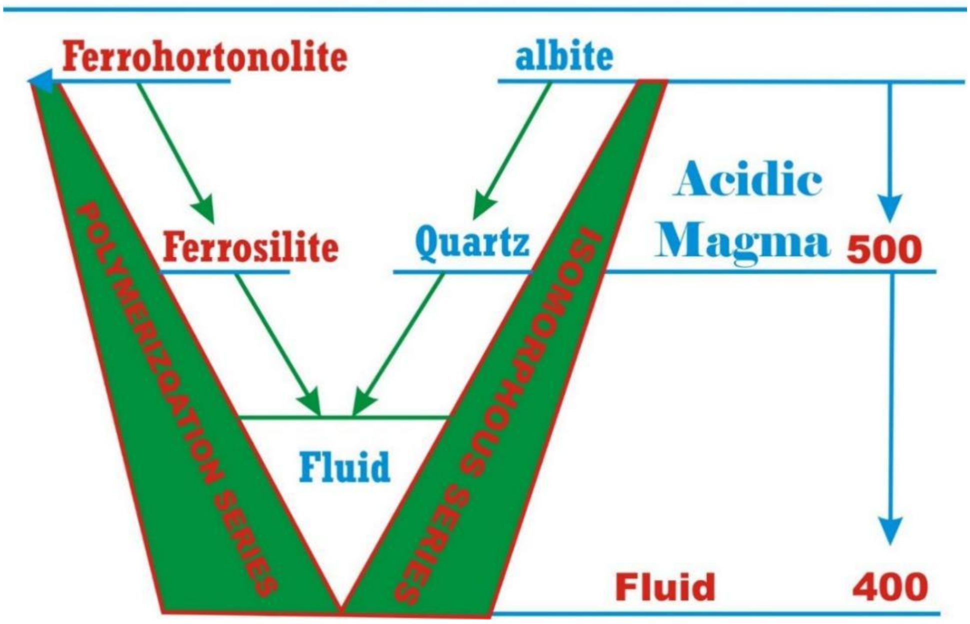
Therefore, ‘p’ and ‘n’ are the two roots of the heterogeneous formed crystal, hence weakly stable, therefore the roots of the heterogeneous crystal, n: p = (6, 2).

F1



Concept 6

F2



G. Concept 7

- For polymerization process, $b_n \longrightarrow z_n$, as n = 7, as shown in concept 1 and Table (1) above. Therefore for polymerization reaction ‘n’ has specific value from the beginning to the end of crystallization. Therefore, ‘n’ = 7, is the only one root of the homogeneous formed crystal, hence strongly stable according to Adams and Bash.

- For isomorphous process and for a given specific value of ‘n’, there is always a unique value of ‘p’ as shown in Table (2) above, Concept 7.

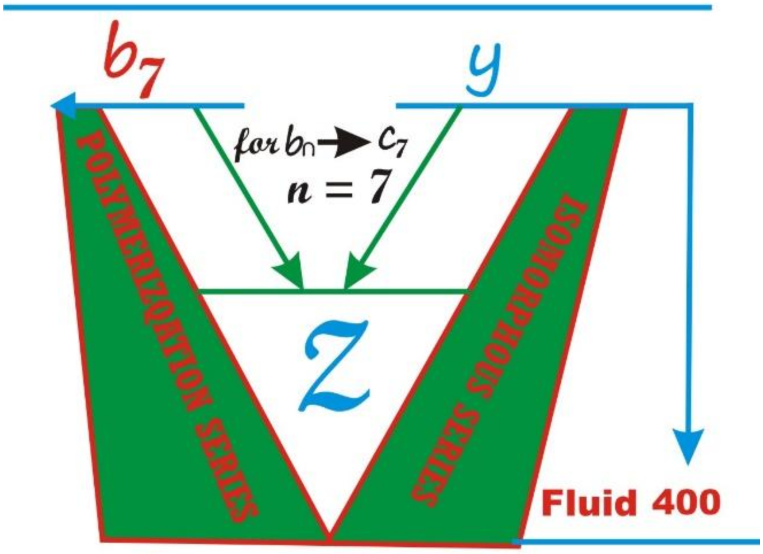
Therefore for, $b_7 \longrightarrow b_1$, as n = 7 and p = 1

Then, $b_n \longrightarrow b_p$; = $b_7 \longrightarrow b_1$; this means that there is continuous change of n to p during crystallization by ionic substitution of ‘n’ by ‘p’ in solid solution as given in concept **G1** and concept **G2** below**.**

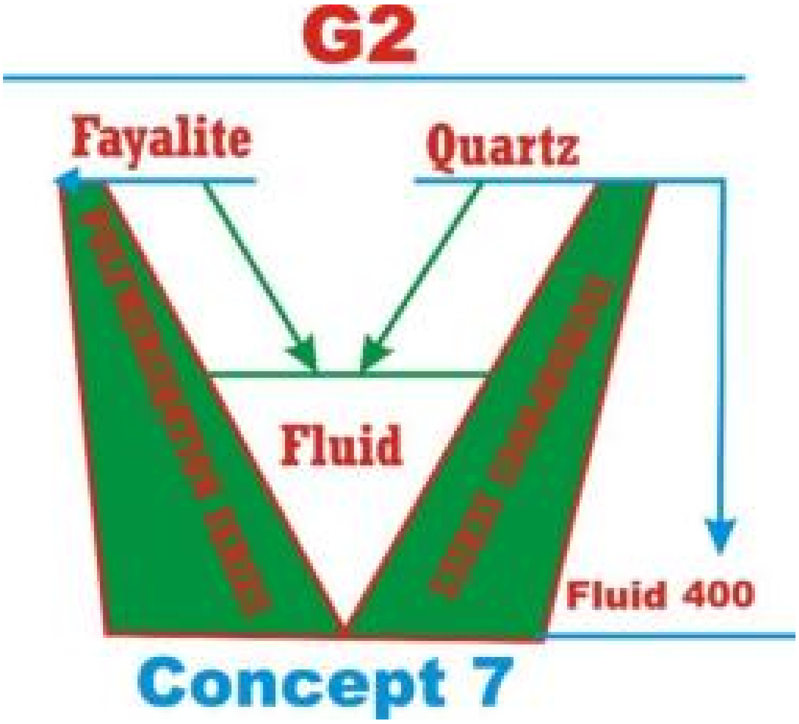
Therefore, ‘p’ and ‘n’ are the two roots of the heterogeneous formed crystal, hence weakly stable, therefore the roots of the heterogeneous crystal are n: p (7, 1).

In mathematical context n:p (1, 7) = p:n (7, 1) is proven to be isomorphic for olivine minerals and the process can be applied on the pyroxene, amphibole and mica minerals throughout the time of crystallization.

G1



Concept 7



4.5 Thermodynamic Explanation of *Basic*, *Acidic* and *Alkali* Magmas and the Evolution of the Minerals and Rocks from the Magmas

Basic magma β_m , is an *Unpolymerized* magma which produces minerals that are unstable in the presence of free silica under thermodynamic change. Because of the instability of these minerals in basic magma β_m in the presence of free silica, the minerals produced from the basic magma β_m during the reaction are called *Basic minerals* e.g., Forsterite in olivine, Enstatite in pyroxene, Kupfferite in amphibole and Phlogopite in mica . Under thermodynamic change forsterite, enstatite, kupfferite and phlogopite are unstable in the presence of excess silica. Unpolymerized magma is a magma that is deficient in silica and usually called basaltic magma in petrology which is about 40-49% silica. Therefore it correct to say in this research that, Basaltic magma is basic in composition. Minerals rich in magnesium are strongly stable because it has one root that is greater than one, but it is metastable under thermodynamic change because minerals rich in magnesium are not in their minimum Gibbs free energy state in the presence of free silica [11].

In conclusion, both basic and acidic minerals can be defined in mineralogy, petrology and crystallography in terms of thermodynamic principle guiding the production or evolution of rock forming minerals from the beginning to the end of crystallization, that is, their stability and instability in the presence of excess silica, which give the genetic make-up of the minerals their names, the "*Basic*" mineral and then the "*Acidic*" mineral. Therefore *"thermodynamic is responsible for the evolution of any minerals and rocks from the melt* $\mathfrak{W}$ *, and the names of the minerals and rocks are related to the thermodynamic change"*. Under thermodynamic change, minerals are either stable or unstable with respect to silica polymorphs such as cristoballite, tridymite and quartz as earlier states in Figures 1, and 2. In the presence of cristoballite in basic magma both forsterite in olivine and anorthite in plagioclase feldspar are unstable under thermodynamic change, but stable if there is no thermodynamic change, and the minerals are basic mineral and the rock is called basic rock known as Basaltic rock. Therefore the stability limit of any mineral in basaltic rock is measured by cristoballite. In the presence of tridymite in intermedite magma both hyalosiderite in olivine and andesine in plagioclase feldspar are either stable or unstable under thermodynamic change, and the minerals are intermediate minerals and the rock is called intermediate rock known as Andesitic rock. Therefore the stability limit of any mineral in Andesitic rock is measured by tridymite. In the presence of Quartz in acidic magma both Fayalite in olivine and albite in plagioclase feldspar are stable under thermodynamic change, but remain stable if there is no thermodynamic change, and the minerals are acidic or alkali minerals and the rock is called acidic or alkali rock known as Rhyolitic rock. Therefore the stability limit of any mineral in rhyolitic rock is measured by quartz. The different between the structures of these silica polymorphs is the way in which silica tetrahedron units are structurally arranged under thermodynamic change.

In this case the evolution of minerals and rocks under thermodynamic change is defined by the stability of silica polymorphs as shown in Figures 1 and 2, as well as Figures 7, and 8.

Comments on the Analogies and the Consequences of the stability of the Minerals under Thermodynamic Change

For each order of crystallization during polymerization process from the beginning to the end of crystallization, there is no change in numerical value under thermodynamic change, that is when one mineral polymerizes to the next mineral, numerical value remains the same within the same order (e.g. from $b_n \longrightarrow k_n$, as n = 1). This means for polymerization process, there is no site for heterogeneous nucleation, that is chemical composition remains the same since the site in the crystal formed is still homogeneous, because it involves only one atom to enter into the lattice of growing crystal Z_0 for a chemical process to take place during crystallization, hence the product has only one root as shown in equation (3) and (4) above since there is no change in chemical composition. Methods that satisfy the root condition and have Z = 1 or 2 as the only root of the characteristics equation of magnitude one or more are called strongly stable according, *Adams,* and *Bash forth*

For each fraction during isomorphuos series, there is continuous change in numerical value from the beginning to the end of crystallization under thermodynamic change that is when one mineral substitutes for one another to form the next mineral, numerical value changes (e.g. from $b_n \longrightarrow b_p$, as n = 1 ⟶7, for all isomorphous process) and this creates room for heterogeneous nucleation because of the addition of another atom to the site of formed homogeneous crystal. This means for isomorphous process, there is a site for heterogeneous nucleation, because it involves two atoms for chemical process to take place during crystallization in the lattice of a formed crystal **Z**, hence the product has two roots. These two atoms of different identities substitute themselves in the already formed site depending on which atom from the pool of competing atoms is closest to the nucleus of the atom in the already formed site which changes the composition of the site but still retains structural identity of the site as shown in equations (9, 11, 12). Methods that satisfy the root condition and have more than one distinct root with magnitude one or more are called weakly stable according to *Adams,* and *Bash forth*

Table 3. Order of Crystallization and Mapping of Minerals using Bowens and Goldschmidt Concepts

Tem (^{0}C)	**Order of Crystallization**	**Bowens Index number ( )**	1	2	3	4	5	6	7
		FELSIC SERIES.	**MAFIC SERIES**						
		PLAGIOCLASE FELDSPAR	**OLIVINE**	**PYROXENE**	**AMPHIBOLE**	**BLACK MICA**	**ALKALINE FELDSPAR**	**WHITE MICA**	**SILICA**
2000	**1**	**Anorthite**	**Forsterite**	**X**	**X**	**X**	**x**	**x**	**x**
1000	**2**	**Bytownite**	**Mgnesio-chrysolite**	**Enstatite**	**X**	**X**	**x**	**x**	**x**
900	**3**	**Labradorite**	**Chrysolite**	**Mnesio-hypersthene**	**Kupfferite**	**X**	**x**	**x**	**x**
800	**4**	**Andesine**	**Hyalosiderite**	**Hypersthene**	**Magnesio-Anthophyllite**	**Phlogopite**	**x**	**x**	**x**
700	**5**	**Oligoclase**	**Hortonolite**	**Ferro-hypersthene**	**Anthophyllite**	**Biotite**	**Orthoclase**	**x**	**x**
600	**6**	**Albite**	**Ferr-hortonolite**	**Eulite**	**Cummingtonite**	**Ferro-biotite**	**Perthite**	**Muscovite**	**x**
500	**7**	**Quartz**	**Fayalite**	**Ferrosilite**	**Grunerite**	**Lepidomelane**	**Albite**	**Paragonite**	**Quartz**

4.6 A new Model showing Mappings and Orders of Crystallization of Materials from the Recharged Magma $\mathfrak{W}$

The Figure (6), below shows the mapping of magma from the beginning to the end of crystallization that is from the basic magma to the acidic magma as mathematically analyzed and mapped above. The complex plain of magma $\mathfrak{W}$, consists of the vertical columns and the rows. Each column represents a fraction called *magma fractionation* as shown in Figure 6 bellow. Each row represents an order called order of crystallization. In this case both magma fractionation and order of crystallization are called *fractional crystallization* as shown in Figure 6 bellow. Ly Mathematically, vertical column represents vertical axis and the row represents the horizontal axis.

- Horizontal axis: b_n, c_n, d_n, f_n, g_n, h_n, and k_n
- Vertical axis: a, e, i, o, u, w, and y

Scale:

Vertical axis: 1uint:100^0C

Horizontal axis: 2unit to 100^0C

Temperature Range from the Beginning to the end of Crystallization

- Horizontal axis: 2000^0C to 400^0C
- Vertical axis: 2000^0C to 400^0C

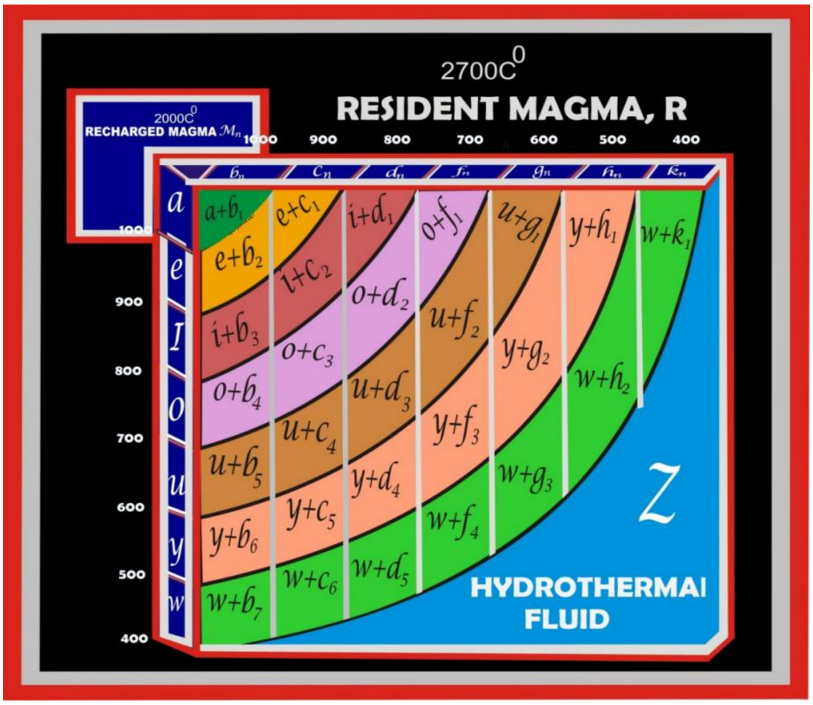


Figure 4. Mapping of Magma with Respect to Gibbs Energy and Le-Chaterlier Principle under Thermodynamic Change

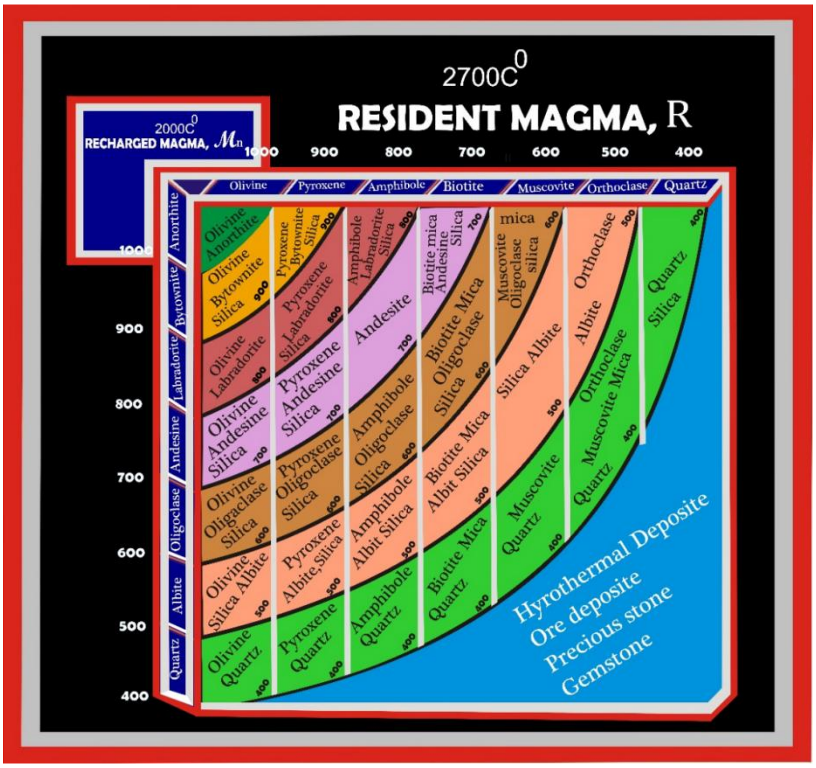


Figure 5. Model of Order of Crystallization and Mapping of Minerals under thermodynamic change

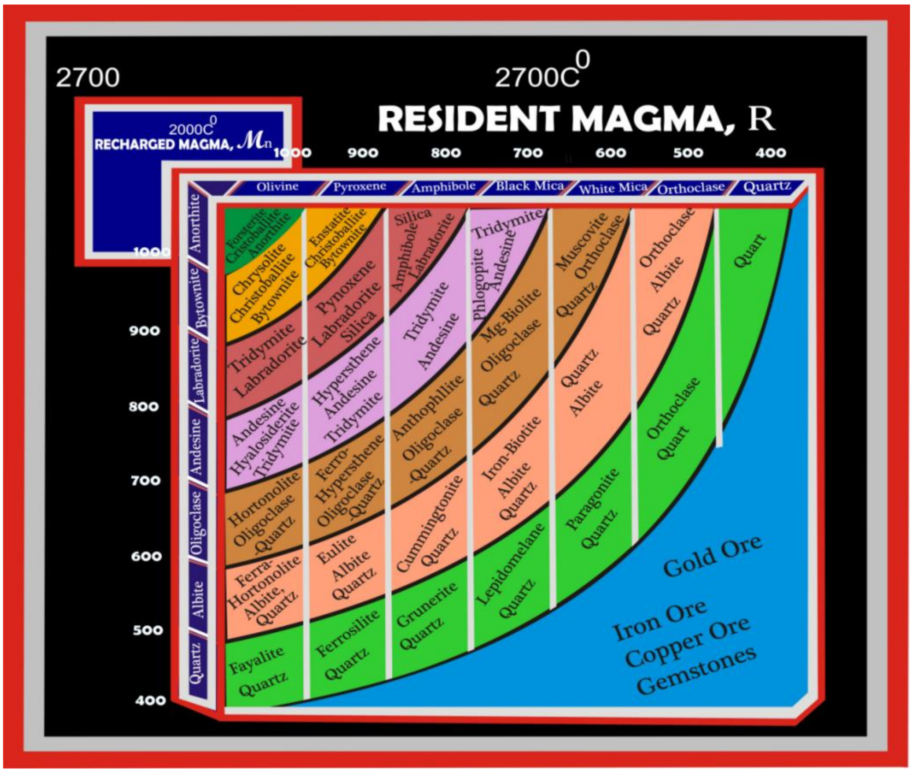


Figure 6. Model of Order of Crystallization and Mapping of Minerals under thermodynamic change

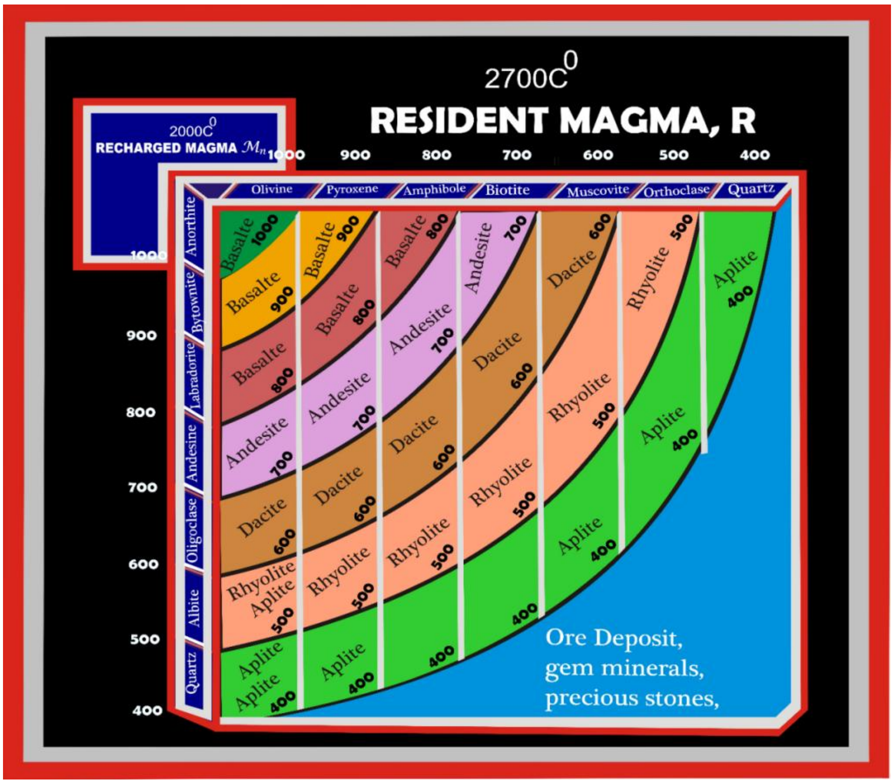


Figure 7. Model of Order of Crystallization and Mapping of Rocks under thermodynamic change

4.7 Modification of Bowens Reaction Series of Rock Forming Minerals

Bowen's uses temperature under isobaric condition to explain sequences of crystallization in which all silicate minerals are formed from the melt as temperature drops from the beginning to the end of crystallization. From this, Bowen's propounded his two reactions series which indicate discontinuous and continuous reactions series where discontinuous reaction series represents mafic minerals and continuous reaction series represents felsic minerals as shown Figure (10) below;

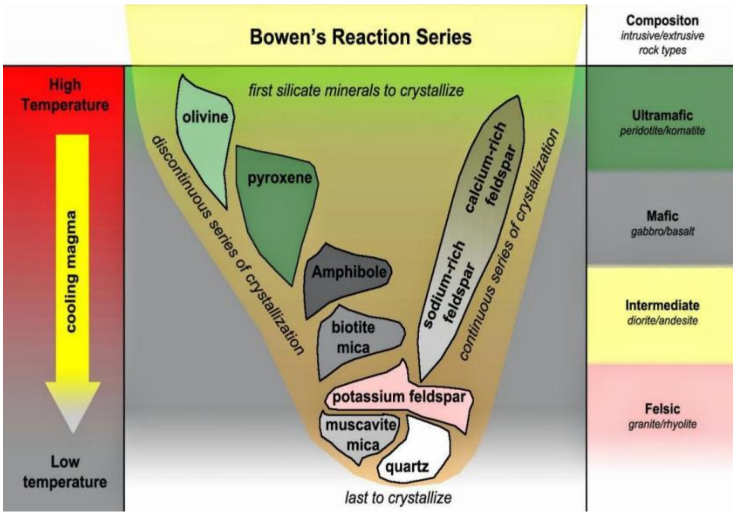


Figure 8. Bowen's Reaction Series

This research uses mathematical mapping method under thermodynamic change to explain sequences of crystallization from the beginning to the end of crystallization and from this, two reactions series have been propounded which modify *Bowen's reactions series*. These two reactions series are *Polymerization* and *Isomorphous* reactions series as shown in Figure (9) and the analogies are described below;

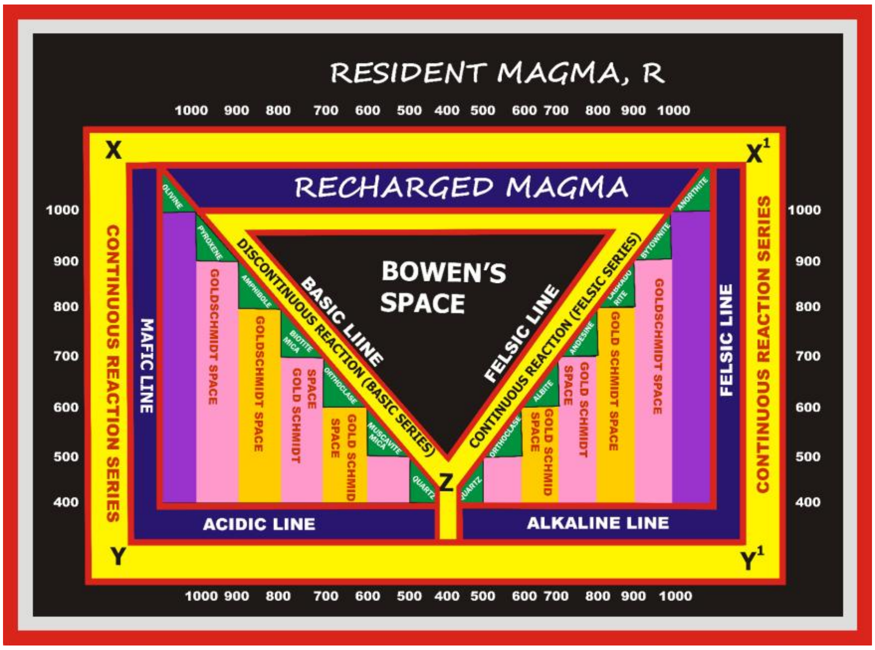


Figure 9. A Model representing Bowen's and Goldschmidt combined Concept

4.8 Bowens and Goldschmidt Combination Model of Minerals and Rocks

This research classifies these minerals according to Goldschmidt and Bowen's explanation as shown in Table 3 above and Figures 10, 11 and 12 below. In Table 3 above, the numerical numbers 1 to 7 are the Bowen's indices called *Bowen's index* in this research, and minerals progress to the next minerals by polymerization reaction by addition of more silica to the already crystallized minerals in a more silica rich melt. This means that according to Bowen's, with drop in temperature of magma olivine progresses to pyroxene by adding silica to olivine mineral, pyroxene progresses to amphibole by addition of silica to pyroxene mineral, amphibole progresses to mica by addition of silica to amphibole crystal, and so on. After this, Bowen's postulated that the remaining melt at lower temperature is quenched to produce the last crystals such as orthoclase, muscovite and quartz. This progressive reaction is what Bowen's called *discontinuous reaction series*, but for the purpose of this research, it is called *polymerization reaction* because of addition of silica to the already crystallized crystal under thermodynamic change.

Bowen's considers the reaction series of plagioclase feldspar as *Continuous reaction series* because of its continuous exchange of element for one another with drop in temperature. For one element to substitute for one another in the Goldschmidt space, it must follow Bowen's index number 1 to 7 as shown in Table 3 above, such that first Bowen's index number under thermodynamic change, must undergo ionic substitution provided that there is Diadochi in the first formed crystal in the Goldschmidt space.

This first Bowen's index number is *forsterite* of pyroxene with Bowen's Index number 1(one) in the Bowen's series and the '*Forsterite* 'under thermodynamic change, would undergo ionic substitution in the Goldschmidt space to form fayalite as the end member of olivine in the olivine series. Both *forsterite*, wholly magnesium rich olivine and *fayalite*, wholly iron rich are within the olivine series with Bowen's index number 1(one) as shown in Table 3 above, Figures, 10, 11 and 12 below.

The second Bowen's index number is *Enstatite* of pyroxene with Bowen's Index number 2(Two) in the Bowen's reaction series and the '*Enstatite*' under thermodynamic change, would undergo ionic substitution in the Goldschmidt space to form *Ferrosilte* as the end member of pyroxene in the pyroxene series. Both *Enstatite*, wholly magnesium rich pyroxene and *Ferrosilite*, wholly iron rich pyroxene are within the pyroxene series with Bowen's index number 2(two) as shown in Table 3 above, Figures, 10, 11 and 12 below.

The third Bowen's index number is '*Kupfferite*' of amphibole with Bowen's Index number 3(Three) in the Bowen's reaction series and the '*Kupfferite*' under thermodynamic change, would undergo ionic substitution in the Goldschmidt space to form *Grunerite* as the end member of amphibole in the amphibole series. Both *Kufferite*, wholly magnesium rich amphibole and *Grunerite*, wholly iron rich amphibole are within the amphibole series with Bowen's index number 3(three) as shown in Table 3 above, Figures, 10, 11 and 12 below. .

The Fourth Bowen's index number is '*Phlogopite*' of mica with Bowen's Index number 4(Four) in the Bowen's reaction series and the '*Phlogopite*' under thermodynamic change, would undergo ionic substitution in the Goldschmidt space to form *Lepidomelane* as the end member of mica in the mica series. Both *Phlogopite*, wholly magnesium rich mica and *Lepidomelane*, wholly iron rich mica are within the mica series with Bowen's index number 4(four) as shown in Table 3 above, Figures, 10, 11 and 12 below.

Number 5 in the series according to Bowen's is *Orthoclase*, a potassium rich alkali feldspar and has an Index number of 5. Sometimes sodium can substitute for potassium in orthoclase to form the end member, *albite*, a sodium rich alkali feldspar in the alkali series as shown in Table 6, Figures 6, 7, 8 and 9 bellow. Both the potassium rich orthoclase and sodium rich Albite form alkali series and they exist at high temperature as *Perthite - Antiperthite* mix ed feldspar and have separate phase at lower temperature as perthite when orthoclase exceeds albite and *Antiperthite* when albite exceeds orthoclase. This process is called *EXSOLUTION* in rock forming minerals from the alkali melt α_m .

The sixth Bowen's index number is '*Muscovite*' of white mica with Bowen's Index number of 6(six) in the Bowen's reaction series and the '*Muscovite*' under thermodynamic change, would undergo ionic substitution in the Goldschmidt space to form *Paragonite* as the end member of mica in the white mica series. Both *Muscovite*, wholly potassium rich white mica and *Paragonite*, wholly sodium rich white mica are within the white mica series with Bowen's index number 6(six) as shown in Table 3 above, Figures, 10, 11 and 12 below. The process in which minerals formed by substitution of one element for anther using Goldschmidt concept is called ***Isomorphous*** series in this research.

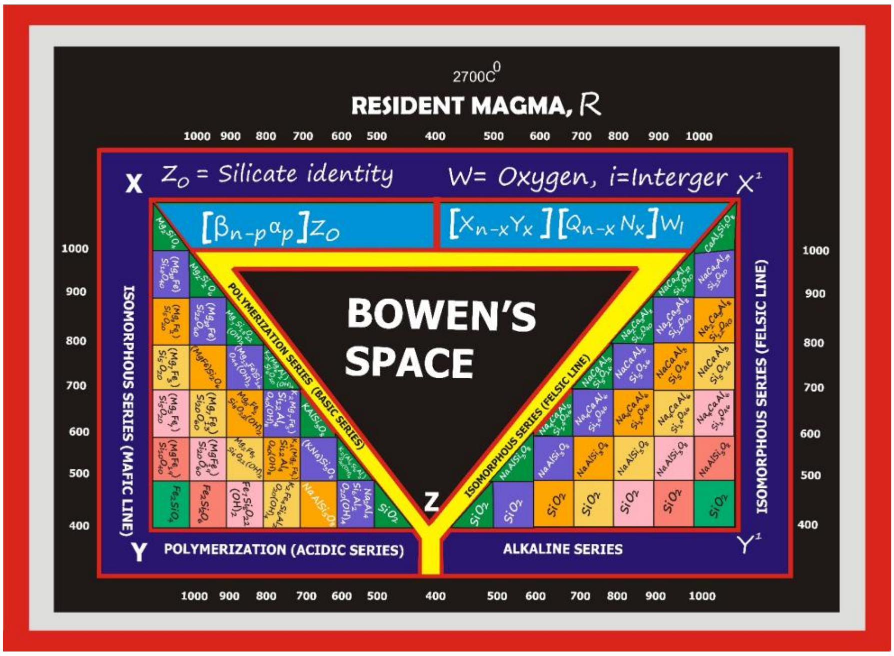


Figure 10. Mathematical Computations of minerals in Bowen and Goldschmidt combined model

- XMZ = Mafic Compositions
- YNZ = Felsic Compositions

Where;

- XMZ AND YNZ = Goldschmidt space
- XYZ = Bowen's space

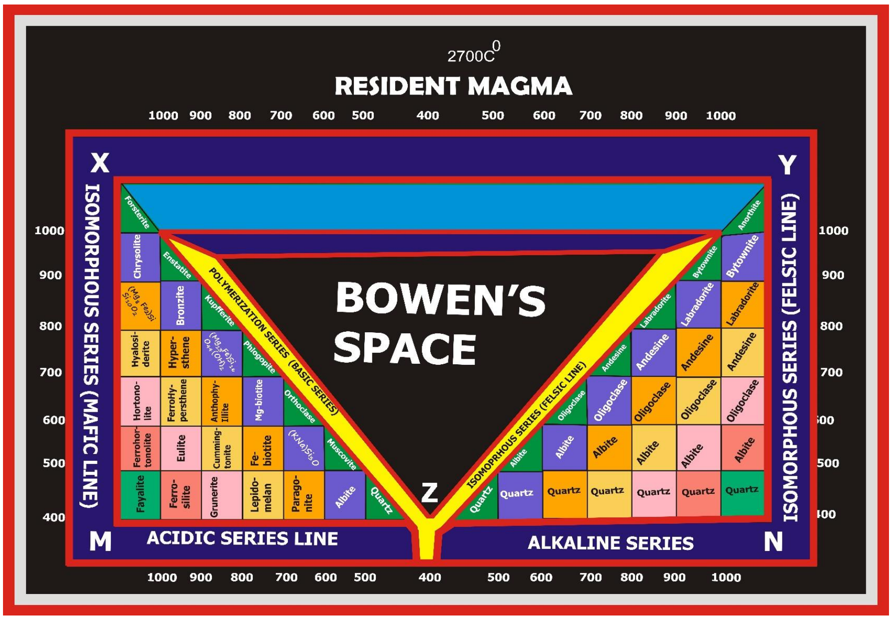


Figure 11. Modification of Bowen's reaction series of Rock forming Minerals using Bowen and Goldschmidt combined model

- XMZ = Mafic Minerals
- YNZ = Felsic Minerals

Where;

- XMZ AND YNZ = Goldschmidt space
- XYZ = Bowen's space

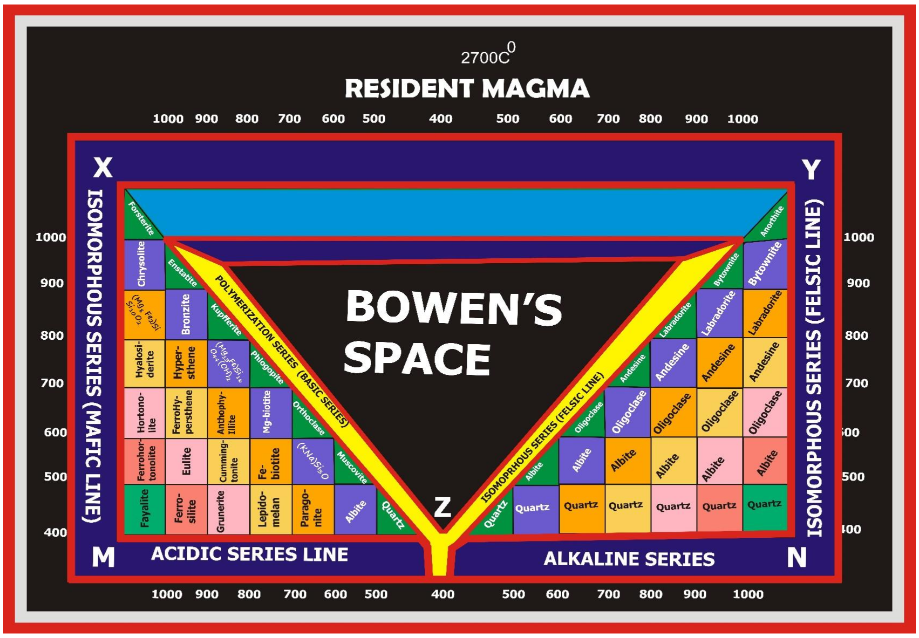


Figure 12. Modification of Bowen's reaction series of Rock forming Minerals using Bowen and Goldschmidt combined model

- XMZ = Mafic Minerals
- YNZ = Felsic Minerals

Where;

- XMZ AND YNZ = Goldschmidt space
- XYZ = Bowen's space

5. Conclusion

In conclusion, when considering the processes of Polymerization and Isomorphous reactions throughout the time of crystallization, therefore, " For spontaneous process under thermodynamic change as temperature changes from the Resident magma $\mathfrak{R}$ to the recharged magmas $\mathfrak{W}$ ($\mathfrak{R} \longrightarrow \mathfrak{W}$), the initially formed crystal β_n , of smaller independent tetrahedron unit of the basic mineral that is, the "Mafic type mineral" in which its $(SiO_4)^{-4}$ independent tetrahedron unit can be linked by the silica tetrahedron $(SiO_4)^{-4}$ by sharing the corners of each adjacent tetrahedron with oxygen atoms in the silica rich melt $\mathfrak{W}$ and then polymerizes to form, new crystal phase β_n of much larger regular structural tetrahedron unit of Basic mineral β_n which is a "Mafic type mineral" that has different form (structure) but similar chemistry (composition) with the initially formed crystal of Basic mineral β_n in the Bowen's space, while simultaneously under thermodynamic change, at the same range of temperature, within its own tetrahedron unit which were initially at equilibrium with the melt $\mathfrak{W} = \mathcal{M}_m + Ƒ_m$, would undergo ionic substitution (isomorphuos reaction) in the Goldschmidt space by substitution of one element for another of similar charge and size to form other new crystal phases of Acidic mineral α_p , that is "a Mafic type mineral" of the same form (structure) of "a definite structure" but different chemistry (composition)" with the initially formed crystals of Basic mineral β_n , provided there is a Diadochi in the crystals of Basic mineral and this is represented mathematically as XMZ and YNZ in Figures 10, 11 and 12 above. Therefore Figures 10, 11 and 12 above modified Bowen's reaction series of rock forming minerals.

6. Contribution to Scientific Knowledge

If all the rules propounded by these scientists such as Goldschmidt, Mendeleev, Ringwood and Wood as well as Bowen to resolve the problems of rock mineral from the magma are valid and conditions controlling these principles are still hold, then the following scientific contributions were made by this research work;

For idealized modified models in this research work,

Polymerization reaction series replaces discontinuous series as described by Bowen's (1928) throughout the time of crystallization and this is shown in Figures, 10, 11 and 12 above.

Isomorphous reaction series replaces continuous reaction series as described by Bowen's (1928), throughout the time of crystallization and this is shown in Figures 10, 11, and 12 above.

The designed models to explain rock forming minerals in this research compliment and modified Bowen's reaction series from the beginning to the end of crystallization under thermodynamic change in which both the *Polymerization* and the *Isomorphous* reactions series are the two reactions series as represented mathematically as XMZ and YNZ in Figures 10, 11, and 12 above.

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[4] BERNARD J. WOOD Department of Earth Sciences, University of Bristol, Bristol BS8 1RJ, UK; b.j.wood@bris.ac.uk Understanding and predicting the behaviour of the chemical elements in the earth's crust, mantle and hydrosphere is a fundamental task in geochemistry

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